

AP CALCULUS AB/BC



QUESTION CATALOGUE

AP Calculus

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1071. What is the domain of the function
 $f(x) = \frac{x^2 - 16}{x^2 + 7x + 12}$?

- (A) $(-4, 4)$ (D) $(-\infty, \infty)$
 (B) $x \neq -3, -4$ (E) $x \neq -4, 4$
 (C) $x \neq -3, -4, 4$

1097. The domain of the function $f(x) = \frac{1}{\sqrt{4-x}}$ is

- (A) $x \geq 0$ (D) $x \geq 4$
 (B) $x > 4$ (E) $x \leq 4$
 (C) $x < 4$

1168. The graph of $y^2 - 5y - 1 = x^2$ is a(n)
 (A) circle (D) line
 (B) ellipse (E) parabola
 (C) hyperbola

1215. If the zeros of $f(x)$ are $x = -2$ and $x = 3$, then the zeros of $f(\frac{x}{2})$ are $x =$

- (A) $-1, \frac{3}{2}$ (D) $-3, \frac{9}{2}$
 (B) $-2, 3$ (E) $-4, 6$
 (C) $-2, 6$

1244. If $f(x) = 3 - x$ and $g(x) = \sqrt{x-5}$, then $f(g(-2)) =$
 (A) 0 (D) $\sqrt{2}$
 (B) $4 - \sqrt{7}$ (E) **Undefined**
 (C) $\sqrt{7}$

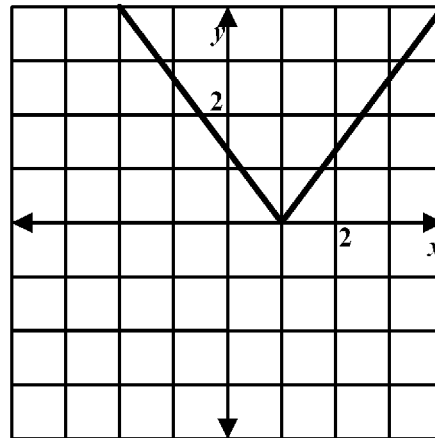
1524. The function $f(x) = \frac{x^2 + 9x - 90}{x^2 + 19x + 60}$ has a removable discontinuity at

- (A) **-15** (D) 7
 (B) -4 (E) 15
 (C) 6

1582. If $f(x) = \frac{3}{x^2 - 2}$ and $g(x) = 4x$, then $g(f(3)) =$

- (A) $\frac{3}{7}$ (D) $\frac{12}{7}$
 (B) $\frac{2}{3}$ (E) $\frac{14}{7}$
 (C) $\frac{12}{3}$

1218.



Which of the following functions is represented by the graph above?

- (A) $f(x) = \left| -\frac{3}{2}x + 1 \right|$ (D) $f(x) = \frac{3}{x}x + 1$
 (B) $f(x) = \left| \frac{3}{2}x + 1 \right|$ (E) $f(x) = \frac{3}{2}x - 1$
 (C) $f(x) = -\frac{3}{2}x + 1$

1525. The function $h(x) = \frac{f(x)}{g(x)}$ is discontinuous whenever

- (A) $g(x) = 1$ (D) $g(x) = 0$
 (B) $g(x)$ is negative (E) $f(x) = g(x)$
 (C) $f(x) = 0$

1726. A polynomial has a relative maximum at $(-5, 4)$, a relative minimum at $(1, -7)$ and a relative maximum at $(4, 2)$, and no other critical points. How many real roots does the polynomial have?

- (A) 2 (D) 5
 (B) 3 (E) 6
 (C) 4

II. DERIVATIVES

1. Definitions

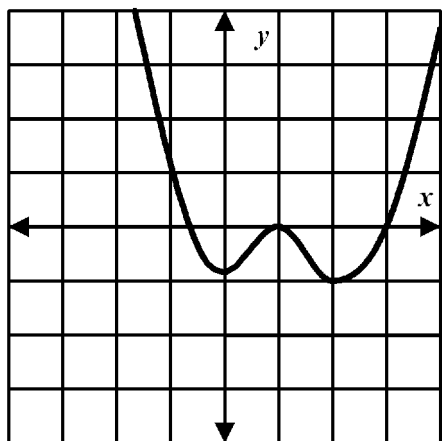
If the graph of $f(x)$ is symmetric with respect to the line $y = x$ and is differentiable everywhere, then the graph of $f'(x)$ is

- (A) Symmetric with respect to the line $y = 0$
(B) Symmetric with respect to the line $x = 0$
 (C) A parabola
 (D) A hyperbola
 (E) None of the above

If $f(x)$ is a continuous differentiable function and $f'(a) = 0$, what can always be said about the graph of $f(x)$ at a ?

- I. The line tangent to the curve at a is horizontal.
 II. The curve is at a local maximum or minimum
 III. There is an inflection point at a .
- (A) I only (D) II only
 (B) I and II (E) III only
 (C) II and III

Base your answers to questions 1241 through 1238 on the graph below of $f(x)$.



Where is $f'(x)$ undefined?

1241. (A) $x = 1$ (D) All of the above
 (B) $x = 2$ (E) None of the above
 (C) $x = 0$

A. Geometric Definition

1. Geometric Definition

Where is $f'(x) < 0$?

1240. (A) $(0, 1)$ and $(2, \infty)$ **(D) $(-\infty, 0)$ and $(1, 2)$**
 (B) $(-\frac{1}{2}, 3)$ (E) $(-\infty, -\frac{1}{2})$ and $(\frac{3}{2}, \infty)$
 (C) $(\infty, 0)$ and $(1, 2)$

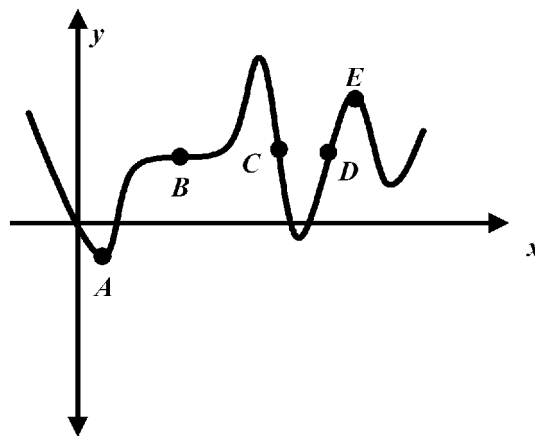
Where is $f'(x) > 0$?

1239. (A) $(\infty, 0)$ and $(1, 2)$ (D) $(-\infty, -\frac{1}{2})$ and $(3, 0)$
(B) $(0, 1)$ and $(2, \infty)$ (E) $(-\infty, -\frac{1}{2})$ and $(\frac{3}{2}, \infty)$
 (C) $(0, 3)$

Where is $f'(x) = 0$?

1238. (A) $x = 1$ (D) $x = 0$ and $x = 2$
 (B) $x = 2$ (E) $x = 0, x = 1, x = 2$
 (C) $x = 0$

Base your answers to questions 607 through 605 on the graph below of $f(x)$.



607. At which point is $f'(x) = 0$ and $f''(x) > 0$?

- (A) A (D) D
 (B) B (E) E
 (C) C

606. At which point is $f'(x) < 0$ and $f''(x) = 0$?

- (A) A (D) D
 (B) B (E) E
 (C) C

1827. There are two lines through the point $(1, -2)$ that are tangential to the parabola $y = x^2 + 2$. Find the x -coordinates of these points of intersection.

- (A) $x = -0.873, 6.873$ (D) $x = -0.765, 1.459$
 (B) $x = -1.236, 3.236$ (E) $x = -1.427, 3.683$
 (C) $x = -2.327, 3.327$

1722. If $2x - 5y = 23$ is the equation of the line normal to the graph of f at the point $(14, 1)$, then $f'(14) =$

- (A) $\frac{2}{5}$ (D) -5
 (B) $\frac{5}{2}$ (E) $-\frac{5}{2}$
 (C) $-\frac{2}{5}$

1662. The line tangent to the graph of $y = -x^{-2}$ at the point $(1, -1)$ intersects both x and y axes. What is the area of the triangle formed by this tangent line and coordinate axes?

- (A) 1 (D) 3.5
 (B) 2 (E) 0.75
 (C) 2.25

1658. The equation of the normal line to the curve $y = 3x^2 - 2x$ at $x = 1$ is

- (A) $y = \frac{x}{4} + \frac{5}{4}$ (D) $y = 4x - 3$
 (B) $y = \frac{x}{4} + \frac{3}{4}$ (E) None of the above
 (C) $y = -\frac{x}{4} + \frac{5}{4}$

1646. What is the slope of the line tangent to the curve $x^2 + y^2 = 9$ when $x = 3$?

- (A) -1 (D) π
 (B) 0 (E) **Infinite slope**
 (C) 1

1645. What is the length of the chord of the circle $x^2 + y^2 = 9$ formed by the line normal to the circle at $x = \frac{3}{2}$?

- (A) 3 (D) 12
 (B) 6 (E) $\sqrt{3}$
 (C) 9

1580. What is the slope of the line tangent to the curve $y = e^x$ at $x = e$.

- (A) e^π (D) e^e
 (B) 1 (E) e^x
 (C) e

1579. Find the equation to the line tangent to the polar curve $r = 2\cos(\frac{1}{2}\theta)$ at $(0, \sqrt{2})$.

- (A) $y = -x + \sqrt{2}$ (D) $y = -\frac{1}{2}x$
 (B) $y = -\frac{1}{2}x + \sqrt{2}$ (E) $y = -2x$
 (C) $y = \sqrt{2}$

1506. Find the equation of the line normal to the curve $x^2 + y^2 = 1$ at $(\cos \frac{5\pi}{4}, \sin \frac{5\pi}{4})$.

- (A) $y = x$ (D) $y = -2x$
 (B) $y = -x$ (E) $y = -x - 2$
 (C) $y = 2x$

1499. Which of the following is the equation of the line tangent to the curve $f(x) = 2\sin(x)$ at $x = \pi$.

- (A) $y = -2x + 2\pi$ (D) $\frac{y}{2} = x + \pi$
 (B) $\frac{y}{2} = -x - \pi$ (E) None of the above
 (C) $y = 2x - 2\pi$

1485. The tangent line to the curve $y = x^3 - 2x + 4$ at the point $(0, 4)$ has an x -intercept at

- (A) $(0, 0)$ (D) $(3, 0)$
 (B) $(-2, 0)$ (E) $(-3, 0)$
 (C) **$(2, 0)$**

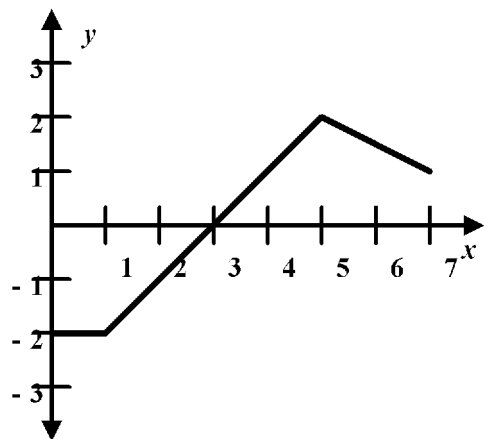
1039. If $f(x) = 3x^2 - 5$, what is the equation for the tangent line to the curve when $x = 1$?

- (A) $y + 2 = 6(x - 1)$ (D) $y - 2 = 6(x + 1)$
 (B) $y - 2 = -12(x - 1)$ (E) $y = 6x$
 (C) $y - 2 = 6(x - 1)$

982. If $f(x) = 2x^3 - 4x^2 - 6x + 1$, then the equation for the normal line to the curve perpendicular to $y + 3 = 2(x - 1)$ is

- (A) $y + 2x + 15 = 0$ (D) $2y - x + 15 = 0$
 (B) $y + 2x - 15 = 0$ (E) $2y + x + 15 = 0$
 (C) **$y - 2x + 15 = 0$**

Base your answers to questions 586 through 584 on the graph below, which shows the velocity of an object moving along a straight line during the time interval $0 \leq t \leq 7$.



586. At what time(s) does the object change direction?

- (A) $t = 3$ and $t = 5$ (D) $t = 6$
 (B) $t = 5$ and $t = 6$ (E) $t = 5$
 (C) $t = 3$

584. At what time(s) does the object reach its maximum acceleration?

- (A) $1 < t < 5$ (D) $t = 6$
 (B) $0 < t < 2$ (E) $t = 5$
 (C) $t = 3$

519. The displacement from the origin of a particle moving on a line is given by $s = t^4 - 2t^3$. The maximum displacement during the time interval $-1 \leq t \leq 3$ is approximately

- (A) 13.966 (D) 18
 (B) 18.685 (E) **34.604**
 (C) 27.000

Base your answers to questions 504 through 500 on the information below.

A particle moves along a horizontal line and its position at time t is $s = \frac{1}{2}t^4 - 2t^3 + 2t^2 + 5$.

504. The position of the particle is always increasing for

- (A) $t > 0$ (D) $t < 0$
 (B) $t > 1$ (E) $0 < t < 2$
 (C) $t > 2$

502. The velocity is increasing when

- (A) $t < 0$
 (B) $t > 2$
 (C) $0 < t < 1$ or $t > 2$
 (D) $\frac{\sqrt{3}-3}{3} < t < \frac{\sqrt{3}+3}{3}$

- (E) $t < -\frac{\sqrt{3}-3}{3}$ and $t > \frac{\sqrt{3}+3}{3}$

500. The particle is at rest when t is equal to

- (A) 0 (D) 0 and 2
 (B) 1 and 2 (E) **0, 1, and 2**
 (C) 0 and 1

Base your answers to questions 499 through 496 on the information below.

The position of a particle moving along a straight line is given by $s = t^3 - 9t^2 + 24t - 6$.

499. The speed of the particle is decreasing for

- (A) $t < 1$ (D) $t < 1$ and $t > 2$
 (B) $t > 2$ (E) all t
 (C) $t < 3$

498. The acceleration is positive

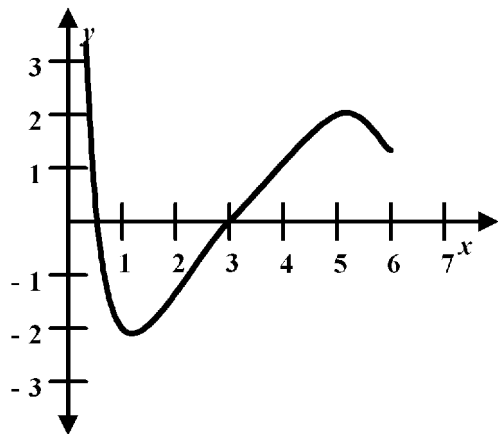
- (A) when $t < 3$ (D) for $2 < t < 4$
 (B) **when $t > 3$** (E) for all $t, t \neq 3$
 (C) for $2 < t < 3$

775. The sides of an equilateral triangle are increasing at the rate of 12 in/sec. How fast is the triangle's area increasing when the sides of the triangle are each 2 inches long?
- (A) $6\sqrt{3}$ ft/sec (D) $36\sqrt{3}$ ft/sec
 (B) $12\sqrt{3}$ ft/sec (E) $48\sqrt{3}$ ft/sec
 (C) $24\sqrt{3}$ ft/sec
740. A boat is being pulled toward a dock by a rope attached to its bow through a pulley on the dock 3 feet above the bow. If the rope is hauled in at a rate of 5 ft/sec, how fast is the boat approaching the dock when its 4 feet from the dock?
- (A) 2 ft/sec (D) 5 ft/sec
 (B) 3 ft/sec (E) 6 ft/sec
 (C) **4 ft/sec**
716. An inverted conical container has a diameter of 36 in and a depth of 12 in. If water is flowing out of the vertex of the container at a rate of 15π in³/sec, how fast is the depth of the water dropping when the height is 3 inches?
- (A) $\frac{15}{2}$ in/s (D) **$\frac{20}{27}$ in/s**
 (B) $\frac{15}{4}$ in/s (E) $\frac{20}{27}\pi$ in/s
 (C) $\frac{10}{9}$ in/s
597. A foot tall rectangular box's length is increasing at the rate of 0.25 feet/sec and its width is decreasing at 0.5 feet/sec. When the length is 0.8 feet and its width is 1 foot, the volume of the box is changing at
- (A) -0.25 ft³/sec (D) 0.25 ft³/sec
 (B) **-0.15 ft³/sec** (E) 0.4 ft³/sec
 (C) 0.15 ft³/sec
555. The height (h) and radius (r) of a cylinder both increase at the rate of 1 in/sec. How fast does the surface area increase?
- (A) $2\pi(r+h)$ (D) $4\pi+2\pi h$
 (B) **$2\pi(3r+h)$** (E) $4\pi r$
 (C) 6π
553. An inverted circular cone with its vertex down has a depth of 10 in and a radius at the top of 4 in. Water is leaking out so that the water level is falling at the rate of 1 in/hr. How fast is the water leaking out of the cone when the water is 6 in deep?
- (A) 1.92π in³/hr (D) 3.8π in³/hr
 (B) 2.4π in³/hr (E) **5.76π in³/hr**
 (C) 3π in³/hr
551. A spherical balloon is being filled with helium at the rate of 9 ft³/min. How fast is the surface area increasing when the volume is 36π ft³?
- (A) 2 ft²/min (D) 3π ft²/min
 (B) 2π ft²/min (E) **6 ft²/min**
 (C) 3 ft²/min
470. A 5-foot-tall person is walking at a rate of 3 ft/sec away from a street lamp that is 16 feet tall. How fast is the length of her shadow changing?
- (A) $\frac{3}{11}$ ft/sec (D) 11 ft/sec
 (B) **$\frac{15}{11}$ ft/sec** (E) 15 ft/sec
 (C) 3 ft/sec
468. Milk spilled from a carton spreads in a circle whose circumference increases at a rate of 20 ft/sec. How fast is the area of the spill increasing when the circumference of the circle is 81π ft?
- (A) **810 ft²/sec** (D) 180 ft²/sec
 (B) 360 ft²/sec (E) 90 ft²/sec
 (C) 270 ft²/sec
212. [Calculator] The best approximation to the increase in volume of a sphere when the radius is increased from 2 to 2.1 is
- (A) 9 (D) 6
 (B) 8 (E) **5**
 (C) 7

II. DERIVATIVES

4. Applications

Base your answers to questions 538 through 537 on the graph below of $f'(x)$.



538. f has a point of inflection at $x =$

- (A) 0.5 (D) 0.5 and 3
 (B) 3 (E) 1 and 5
 (C) 5

537. f has a local minimum at $x =$

- (A) 0 (D) 3
 (B) 0.5 (E) 5
 (C) 1

480. On the closed interval $[0, 2\pi]$, the maximum value of the function $f(x) = 5\sin x - 12\cos x$ is

- (A) 0 (D) 12
 (B) $\frac{60}{13}$ (E) 13
 (C) 5

225. The minimum value of the slope of the curve $y = x^6 - x^4 + 3x$ is

- (A) -3.025 (D) 2.595
 (B) -2.595 (E) 3.025
 (C) 0

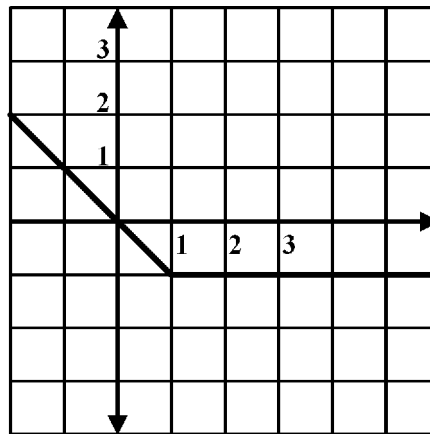
215. If f is differentiable and difference quotients underestimate the slope of f at $x = a$ for all $h > 0$, which of the following must be true?

- (A) $f'' < 0$ (D) $f'' > 0$
 (B) $f' < 0$ (E) None of the above
 (C) $f' > 0$

D. Maximum, Minimum and Inflection Point

1. Maximum, Minimum and Inflection Point

Base your answers to questions 320 through 318 on the graph below of $f'(x)$.



320. The function is concave downward for which interval?

- (A) (1, 2) (D) (-1, 1)
 (B) (1, 4) (E) (-1, 4)
 (C) (2, 3)

319. Which statement best describes f at $x = 0$?

- (A) f is a minimum
(B) f is a maximum
 (C) f has a root
 (D) f has a point of inflection
 (E) None of the above

318. Which of the following is true based upon the graph above?

- (A) f has a local maximum at $x = 0$**
 (B) f has a local maximum at $x = -1$
 (C) f is a constant for $1 < x < 4$
 (D) f is decreasing for $-1 < x < 1$
 (E) f is discontinuous at $x = 1$

151. The number of inflection points of $f(x) = 4x^4 - 4x^2$ is

- (A) 0 (D) 3
 (B) 1 (E) 4
(C) 2

Given the function $f(x) = e^{-x}(x^2 + 1)$

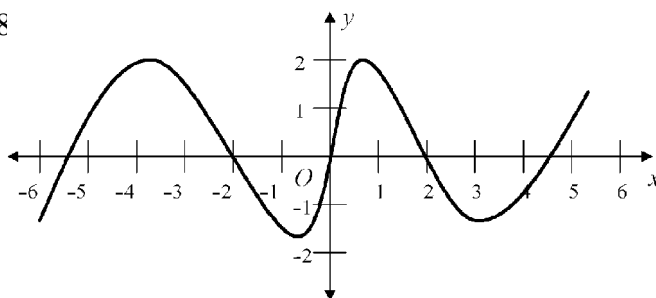
1271. (a) For what values of x is f increasing?
 (b) For what values of x is f concave up?
 (c) Set up, but do not evaluate an integral (or set of integrals) which give the arc length of the segment of f from $x = 0$ to $x = 1$.

(a) $\{ \}$

(b) $x < 1$ or $x > 3$

(c) $\int_0^1 \sqrt{e^{-2x}(x-1)^4 + 1} \, dx$

1268



The figure above shows the graph of f' , the derivative of a function f . The domain of f is the set of all real numbers x such that $-6 \leq x \leq 6$.

- (a) For what values of x does the graph of f have a horizontal tangent?
 (b) For what values of x in the interval $-6 < x < 6$ does f have a relative minimum? Justify your answer.
 (c) For what values of x is the graph of f concave upward?

(a) $x = -\frac{11}{2}, -2, 0, 2, \frac{9}{2}$

(b) $x = -\frac{11}{2}, 0, \frac{9}{2}$

(c) $x < -\frac{7}{2}$ or $-\frac{1}{2} < x < 1$ or $x > 3$

Let f be a differentiable function, defined for all real x , with the following properties:

1217. 1) $f''(1) = f'(1) = f(1)$
 2) f is a polynomial of degree at most 2
 3) f has only one fixed point (that is, there is exactly one p such that $f(p) = p$).

Find $f(x)$.

$f(x) = 0$

1266. A particle moves along the x -axis in such a way that its acceleration at time t for $t \geq 0$ is given by $a(t) = 6\sin(3t)$. At time $t = 0$, the velocity of the particle is $v(0) = \sqrt{2}$ and its position is $x(0) = 0$.

- (a) Write an equation for the velocity $v(t)$ of the particle.
 (b) Write an equation for the position $x(t)$ of the particle.

(a) $v(t) = -2\cos(3t) + \sqrt{2} + 2$

(b) $x(t) = -\frac{2}{3}\sin(3t) + t\sqrt{2} + 2t$

1265. A particle moves along the x -axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 1 - 2\cos \pi t$.

- (a) Find the acceleration $a(t)$ of the particle at any time t .
 (b) Find all values of t , $0 \leq t \leq 4$, for which the particle is at rest.
 (c) Find the position $x(t)$ of the particle at any time t if $x(0) = 0$.

(a) $a(t) = 2\pi \sin(\pi t)$

(b) $t = \frac{1}{3}, \frac{5}{3}, \frac{7}{3}, \frac{11}{3}$

(c) $x(t) = t - \frac{2\sin(\pi t)}{\pi}$

Let $f(x) = 8 - 2x^2$ for $x \geq 0$ and $f(x) \geq 0$.

1264.

- (a) The line tangent to the graph of f at the point (x, y) intercepts the x -axis at $x = 3$. What are the coordinates of the point (x, y) ?
 (b) An isosceles triangle whose base is the interval from $(0, 0)$ to $(k, 0)$ has its vertex on the graph of f . For what value of k does the triangle have maximum area? Justify your answer.

(a) $(\sqrt{5} + 3, -12\sqrt{5} - 20)$

(b) $k = \sqrt{\frac{20}{3}}$

729. Using a right Riemann sum, what is the area under the curve $y = x^2 + x$ from $x = 0$ to $x = 3$ when $n = 6$?

- (A) 10.625 (D) 13.625
 (B) 13.438 (E) **16.625**
 (C) 13.500

723. Using the Trapezoidal Rule, what is the area under the curve $y = x^2 + x$ from $x = 0$ to $x = 3$ when $n = 6$?

- (A) 10.625 (D) **13.625**
 (B) 13.438 (E) 16.625
 (C) 13.500

703. A Riemann sum to calculate the area under $f(x)$ on the interval $a \leq x \leq b$ with an infinite number of subintervals will yield the value

(A) $\int_a^b f(x)dx$ (D) $\left. \frac{df(x)}{dx} \right|_{(b,a)}$

(B) $f(b) - f(a)$ (E) $\frac{d}{dx} \int_a^b f(x)dx$

(C) $\left. \frac{df(x)}{dx} \right|_{(a,b)}$

694. Using the Midpoint Formula, what is the area under the curve $y = x^2 + x$ from $x = 0$ to $x = 3$ when $n = 6$?

- (A) 10.625 (D) 13.625
 (B) **13.438** (E) 16.625
 (C) 13.500

682. Using a right Riemann sum, what is the area under the curve $y = 2x - x^2$ from $x = 1$ to $x = 2$ when $n = 4$?

- (A) **0.53125** (D) 0.67187
 (B) 0.65625 (E) 0.78125
 (C) 0.66667

658. What is an approximation for the area under the curve $y = 4x - x^2$ on the interval $[0, 4]$ using the midpoint formula with 20 subintervals?

- (A) **10.589** (D) 10.681
 (B) 10.624 (E) 10.703
 (C) 10.667

657. What is an approximation for the area under the curve $y = \frac{1}{x}$ on the interval $[2, 5]$ using the trapezoidal rule with 9 subintervals?

- (A) 0/868 (D) **0.918**
 (B) 0.915 (E) 0.968
 (C) 0.916

654. What is an approximation for the area under the curve $y = \frac{3}{1+x^2}$ on the interval $[0, 3]$ using the trapezoidal rule with 5 subintervals?

- (A) 2.932 (D) 3.750
 (B) **3.742** (E) 4.552
 (C) 3.747

651. What is an approximation for the area under the curve $y = \sqrt{5+x^5}$ on the interval $[0, 3]$ using the midpoint formula with 4 subintervals?

- (A) 11.992 (D) 17.059
 (B) **16.155** (E) 22.126
 (C) 16.456

647. Using a left Riemann sum, what is the area under the curve $y = \sqrt{x}$ from $x = 1$ to $x = 3$ when $n = 4$?

- (A) 2.976 (D) 2.793
 (B) 2.800 (E) **2.610**
 (C) 2.797

460. Using a left Riemann sum, what is the area under the curve $y = 2x - x^3$ from $x = 1$ to $x = 2$ when $n = 4$?

- (A) 0.53125 (D) 0.67187
 (B) 0.65625 (E) **0.78125**
 (C) 0.66667

1851. If $\int_0^3 (2x^2 - x + 3) dx$ is approximated by three inscribed rectangles of equal width on the x -axis, the approximation is

- (A) 16 (D) 23.5
 (B) 22 (E) 31
 (C) 22.5

Find the area in the first quadrant bounded by the

1795. graphs of $y = \cot x$, $y = 2\sin x$ and the x -axis.

- (A) 0.438 (D) 0.909
 (B) 0.470 (E) **1.571**
 (C) 0.746

The total area enclosed between the graphs of

1784. $y = 2\cos x$ and $y = \frac{x}{2}$ is

- (A) 3.862 (D) **4.812**
 (B) 3.985 (E) 5.914
 (C) 4.547

1772. If $\int_a^b f(x) dx = -2$ and $\int_a^b g(x) dx = -5$, which of the following must be true?

- I. $f(x) > g(x)$ for $a \leq x \leq b$
 II. $\int_a^b [f(x) - g(x)] dx = 3$
 III. $\int_a^b [f(x)g(x)] dx = 10$

- (A) I only (D) II and III
 (B) **II only** (E) I, II and III
 (C) I and II

The area in the first quadrant bounded by the

1714. graphs of $y = x^2 + 3$ and $y = 12$ is

- (A) 9 (D) 36
 (B) **18** (E) 30
 (C) 24

848. What is the average area of all circles with radii between 3 and 6?

- (A) 15π (D) **21π**
 (B) 17π (E) 23π
 (C) 19π

The area of the region bounded by the graph of

1674. $y = 2xe^{x^2}$ and the x -axis from $x = 0$ to $x = 1$ is

- (A) $e^2 - 1$ (D) e
 (B) $e - 1$ (E) $e^3(1 - e)$
 (C) $e^2 - e$

Let $f(x) = x\sin(x - \pi)$, $0 < x < 2\pi$. The total area

1668. bounded by $f(x)$ and the x -axis on the interval $0 < x < 2\pi$ is

- (A) π (D) **4π**
 (B) 2π (E) 5π
 (C) 3π

The area of the region enclosed by $y = 2x^2 - 4$

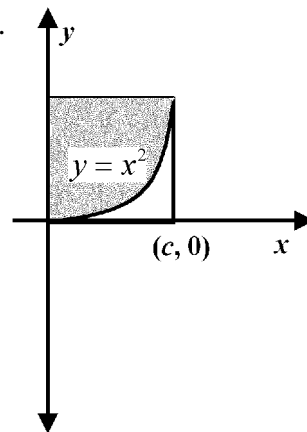
1651. and $y = \sqrt{9 - x^2}$ is

- (A) 8.368 (D) 15.538
 (B) 9.082 (E) **16.736**
 (C) 12.874

847. What is the average area of all squares with sides between 3 and 12?

- (A) **63** (D) 69
 (B) 65 (E) 71
 (C) 67

840.



What is the ratio of the area of the rectangle to the shaded part of it above $y = x^2$?

- (A) 2 : 1 (D) 4 : 3
 (B) 3 : 1 (E) 5 : 4
 (C) **3 : 2**

1670. The base of a solid is the region bounded by the line $-y + 6 = 2x$ and the coordinate axes. What is the volume of the solid generated if every cross section perpendicular to the x -axis is a semicircle?

- (A) $\frac{7}{2}\pi$ (D) 14π
 (B) $\frac{9}{2}\pi$ (E) 18π
 (C) 9π

1661. The region bounded by the graphs of $y = 3 - e^{-x}$ and $y = x^2$ is revolved around the x -axis. The volume of the resulting solid is

- (A) 9.805 (D) 27.362
 (B) 11.473 (E) **30.805**
 (C) 19.975

1141. What is the volume of the solid when the region bounded by $xy = 6$ and $x + y = 5$ is revolved about the x -axis?

- (A) π (D) $\frac{1}{6}\pi$
 (B) $\frac{2}{3}\pi$ (E) $\frac{1}{9}\pi$
 (C) $\frac{1}{3}\pi$

1100. What is the volume of the solid when the region bounded by $y = \frac{x}{3}$, $y = x$, $y = 1$ and $y = 2$ is revolved about the x -axis?

- (A) 8π (D) 12π
 (B) $\frac{28}{3}\pi$ (E) $\frac{40}{3}\pi$
 (C) $\frac{32}{3}\pi$

1076. What is the volume of the solid when the region bounded by $y = \cos x$, $x = \frac{\pi}{2}$ and $y = 0$ is revolved about the y -axis?

- (A) 1 (D) $\frac{1}{2}\pi - 1$
 (B) π (E) **$1 - 2\pi$**
 (C) 2π

1030. What is the volume of the solid when the region bounded by $x = \sqrt{3+y}$, $x = 0$ and $y = 6$ is revolved about the y -axis?

- (A) $\frac{27}{2}\pi$ (D) $\frac{81}{2}\pi$
 (B) $54\pi - \pi\sqrt{3}$ (E) 6π
 (C) $4\pi\sqrt{6} - 2\pi\sqrt{3}$

966. What is the volume of the solid when the region bounded by $y = x^{3/2}$, $x = 0$, $x = 4$ and $y = 0$ is revolved about the x -axis?

- (A) 1028π (D) $\frac{64}{5}$
 (B) **64π** (E) $\frac{256}{5}\pi\sqrt{2}$
 (C) 3π

941. What is the volume of the solid that has a circular base of radius r and every plane section perpendicular to a diameter is semi-circle?

- (A) $\frac{1}{2}\pi r^3$ (D) $\frac{1}{3}r^3$
 (B) $\frac{1}{4}\pi r^3$ (E) $\frac{2}{3}r^3$
 (C) $\frac{1}{3}\pi r^3$

940. What is the volume of the solid that has a circular base of radius r and every plane section perpendicular to a diameter is an isosceles triangle?

- (A) $\frac{1}{2}\pi r^3$ (D) $\frac{4}{3}r^3$
 (B) $\frac{2}{3}r^3$ (E) $\frac{2}{3}\pi r^3$
 (C) πr^3

939. What is the volume of the solid that has a circular base of radius r and every plane section perpendicular to a diameter is an equilateral triangle?

- (A) $r^3\sqrt{3}$ (D) $\frac{4\sqrt{3}}{3}r^3$
 (B) $2r^3\sqrt{3}$ (E) $\frac{8\sqrt{3}}{3}r^3$
 (C) $4r^3\sqrt{3}$

922. What is the volume of the solid when the region bounded by $y = \sqrt{x}$, $y = x$ is revolved about the x -axis?

- (A) π (D) $\frac{1}{3}\pi$
 (B) 2π (E) $\frac{1}{6}\pi$
 (C) $\frac{1}{2}\pi$

917. What is the volume of the solid that has a circular base of radius r and every plane section perpendicular to a diameter is a square?

- (A) $2\pi r^3$ (D) $\frac{8}{3}\pi r^3$
 (B) $4\pi r^3$ (E) $\frac{16}{3}r^3$
 (C) $\frac{8}{3}r^3$

1046. If the population of a city increases continuously at a rate proportional to the population at that time and the population doubles in 20 years, then what is the ratio of the population after 60 years to the initial population?

- (A) 3:1 (D) 9:2
 (B) 3:2 (E) 9:4
 (C) **9:1**

1043. Which of the following curves passes through the point (1,1) and whose slope at any point is equal to y^2x ?

- (A) $y = \frac{2}{3-x^2}$ (D) $y = \frac{3}{2-x^2}$
 (B) $y = \frac{3}{x^2-2}$ (E) $y = \frac{2}{x^2-3}$
 (C) $y = -x^2$

1033. The general solution to the differential equation $dy/dx = 2$ is a family of

- (A) parabolas (D) ellipses
 (B) **straight lines** (E) circles
 (C) hyperbolas

1032. The general solution to the differential equation $dy/dx = x$ is a family of

- (A) **parabolas** (D) ellipses
 (B) straight lines (E) circles
 (C) hyperbolas

1029. If $\frac{dy}{dx} = \sin x \sec y$ and $y = \frac{\pi}{2}$ when $x = 0$, what is the solution to the differential equation?

- (A) $y = \cos^{-1}(\sin x + 2)$ (D) $y = \cos^{-1}(\sin x)$
 (B) **$y = \sin^{-1}(2 - \cos x)$** (E) $y = \sin^{-1}(\cos x)$
 (C) $y = \sin^{-1}(\cos x + 2)$

1027. If $\frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}}$ and $y = 2$ when $x = \sqrt{3}$, what is the solution to the differential equation?

- (A) $y = \sqrt{1+x^2} + 1$ (D) $y = \sqrt{1+x^2}$
 (B) $y = \frac{1}{2}\sqrt{1+x^2} + 1$ (E) $y = 2\sqrt{1+x^2}$
 (C) $y = \frac{1}{2}\sqrt{1+x^2}$

1023. If $\frac{dy}{dx} = \frac{1}{2}e^y$ and $y = 2$ when $x = 2$, what is the solution to the differential equation?

- (A) **$y = \ln|\frac{2}{x}| + 2$** (D) $y = \frac{1}{2}\ln|x| + 2$
 (B) $y = \ln|\frac{x}{2}| + 2$ (E) $y = \ln|x| + 2$
 (C) $y = 2\ln|x| + 2$

1022. If $\frac{dy}{dx} = \frac{y}{\sqrt{x}}$ and $y = 1$ when $x = 1$, what is the solution to the differential equation?

- (A) $y = e^{2\sqrt{x}-2}$ (D) $y = e^{\sqrt{x}} - 1$
 (B) $y = e^{\sqrt{x}}$ (E) $y = e\sqrt{x} - 1$
 (C) $y = \sqrt{x}e^{\sqrt{x}}$

1014. What is a solution to the differential equation $ydy = 2xdx$?

- (A) $x^2 + y^2 = 4$ (D) $x^2 = 2 - y^2$
 (B) $y^2 = x^2$ (E) $x^2 - y^2 = 2$
 (C) **$y^2 - 2x^2 = 0$**

965. A growth rate of 5% per year is equal to a continuous growth rate of

- (A) $\ln(0.95)$ (D) $0.95\ln(1.05)$
 (B) **$\ln(1.05)$** (E) $\ln(1.05) - \ln(0.95)$
 (C) $1.05\ln(0.05)$

1779. A particle moves along a curve so that at any time $t \geq 0$ its velocity is given by $v(t) = \ln(t + 1) - t^2 + 1$. The total distance traveled by the particle from $t = 1$ to $t = 3$ is

- (A) 3.986 (D) 4.697
 (B) 4.289 (E) **4.778**
 (C) 4.508

1724. A particle travels in a straight line with a constant acceleration of 2 meters per second per second (m/s^2). If the velocity of the particle is 5 meters per second at the time $t = 1$ second, how far does the particle travel from $t = 1$ to $t = 3$?

- (A) 7 m (D) 11 m
 (B) 8 m (E) **14 m**
 (C) 10 m

1718. A particle travels along the x -axis with velocity at time t , $v(t) = \cos(t^2)$. If at time $t = 0$ the particle is at $x(0) = 2$, where is the particle at $t = 2$?

- (A) 0.492
 (B) 1.529
 (C) 2.982
 (D) **2.461**
 (E) It cannot be determined from the information given.

1680. A particle moves along a path so that at any time t its acceleration is given by $a(t) = 2t + 1$. At time $t = 0$, its velocity is $v(0) = -6$. For what value(s) of t is the particle at rest?

- (A) 0 only (D) 2 and -3
 (B) 2 only (E) **No values**
 (C) -3 only

1663. A particle moves along a path so that its velocity is given by $v(t) = t^2 - 4$. How far does the particle travel from $t = 0$ to $t = 4$?

- (A) $\frac{16}{3}$ (D) 16.819
 (B) 8 (E) 20
 (C) **16**

1222. The acceleration of a car traveling on a straight track along the x -axis is given by the equation $a(t) = 2t + 1$, where a is in meters per second squared and t is in seconds. If at $x(0) = 0$ and $v(0) = 0$, what is its displacement at $t = 3$?

- (A) 1 m (D) 12 m
 (B) 7 m (E) **13.5 m**
 (C) 9 m

1195. The acceleration of a car traveling on a straight track along the y -axis is given by the equation $a = 5$, where a is in meters per second squared and t is in seconds. If at $t = 0$ the car's velocity is 3 m/s, what is its velocity at $t = 2$?

- (A) 5 m/s (D) **13 m/s**
 (B) 3 m/s (E) 15 m/s
 (C) 10 m/s

1153. The velocity of a particle is given by the equation $v(t) = 3t - 4$. If $s(0) = 2$, then what is the position function of the particle?

- (A) $s(t) = 3t^2 + 2$ (D) $s(t) = 3t^2 - 4t + 2$
 (B) $s(t) = \frac{3}{2}t^2 - 4t + 2$ (E) $s(t) = t^2 - 4t + 2$
 (C) $s(t) = 3$

1091. The velocity of a car traveling on a straight track along the y -axis is given by the equation $v(t) = 12t^2 - 6t + 2$, where v is in meters per second and t is in seconds. The vehicle's initial position is $y = -1$ m. At what time does the car pass the origin?

- (A) 0 s (D) 3 s
 (B) **1 s** (E) 4 s
 (C) 2 s

1080. The equation $v(t) = 3t^2 - 4t + 2$, where v is in meters per second and t is in seconds gives the velocity of a vehicle moving along a straight track. The vehicle's initial position is 3 meters. What distance has the vehicle traveled after 4 seconds?

- (A) 30 m (D) **33 m**
 (B) 31 m (E) 34 m
 (C) 32 m

1184. For time t , $0 \leq t \leq 2\pi$, the position of a particle, is given by $x = \sin^2 t$ and $y = e^t \cos t$.

- (a) Find the formula for the slope of the path of the particle as a function of time.
- (b) For what t is the line tangent to the curve vertical.
- (c) Set up an integral for the distance traveled by the particle from $t = 0$ to $t = 1$.

(a) $s(t) = \frac{e^t (\cos t - \sin t)}{2 \sin t \cos t}$

(b) $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

(c) $\int_0^1 \sqrt{\sin^2 2t + e^{2t} (\cos t - \sin t)^2} dt$

1183. Let f be function given by $f(x) = x^2$ and let g be the function given by $g(x) = kx - 4$, where k is a positive constant such that g is tangent to the graph of f .

- (a) Find the value of k .
- (b) Find the area bounded on top by the line perpendicular to g and on the bottom by $f(x)$.
- (c) Find the volume of the solid generated by revolving the region from part (b) about the line $y = 0$.

- (a) $k = 4$
- (b) **44.667**
- (c) **385.36**

1315. Consider $\int_0^{\frac{\pi}{2}} \frac{1}{1+x^4} dx$.

- (a) Requirite the denominator as $(x^2 + 1)^2 - 2x^2$.
- (b) Split up the denominator into a product of linear factors.
- (c) Integrate by partial fractions.

1172. Let R be enclosed by the graph of $y = x \ln x$, the line $x = 2$, and the x -axis.

- (a) Find the net area of region R .
- (b) Find the volume of the solid generated by revolving region R about the x -axis.
- (c) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated by revolving region R about the line $x = 2$.

(a) $A = 2 \ln 2 - 1$

(b) $V = \pi \left[\frac{8}{3} (\ln 2)^2 - \frac{16}{9} \ln 2 + \frac{14}{27} \right]$

(c) $V = 2\pi \int_1^2 (2-x)x \ln x dx$

1164. Let f be a function that is defined for all real numbers x and that satisfies the following properties.

- i. $f''(x) = 10x - 12$
- ii. $f'(1) = -16$
- iii. $f(0) = 8$

- (a) Find all values of x such that the line tangent to the graph at $(x, f(x))$ is horizontal.
- (b) Find $f(x)$.
- (c) Find the average value of f' on the interval $2 \leq x \leq 5$.

(a) $-6, 3$

(b) $f(x) = \frac{5x^3}{3} - 6x^2 - 9x + 8$

(c) $\frac{232}{3}$

1. Series of Constants

1. Convergence and Divergence

1177. The series $\sum_{n=1}^{\infty} \left(\frac{1}{5^n}\right)$ is

- (A) convergent and decreasing
- (B) convergent and increasing
- (C) divergent and decreasing
- (D) divergent and increasing
- (E) divergent and remain the same

1174. The series $\sum_{n=1}^{\infty} \left(\frac{2^n}{1+2^n}\right)$ is

- (A) neither increasing nor decreasing
- (B) decreasing and convergent
- (C) decreasing and divergent
- (D) increasing and convergent**
- (E) increasing and divergent

Which one of the following series is divergent?

1159. (A) $\sum_{k=1}^{\infty} k^{-\frac{5}{2}}$ (D) $\sum_{k=1}^{\infty} \frac{1}{k^{11}}$
- (B) $\sum_{k=1}^{\infty} \frac{1}{k^{\pi}}$ (E) $\sum_{k=1}^{\infty} k^{-9}$
- (C) $\sum_{k=1}^{\infty} \frac{1}{k}$

1158. The series $\sum_{n=1}^{\infty} \left(\frac{n^n}{n!}\right)$ is

- (A) increasing and convergent
- (B) decreasing and convergent
- (C) increasing and divergent**
- (D) decreasing and divergent
- (E) neither increasing or decreasing

1083. For what values of n is the series $\sum_{n=1}^{\infty} \left(\frac{8^n}{n!}\right)$

decreasing?

- (A) $n \geq 8$ (D) $n \leq 7$
- (B) $n \leq 8$ (E) $n \leq 9$
- (C) $n \geq 7$

Which of the following series is convergent?

1148. (A) $\sum_{n=1}^{\infty} \frac{\ln k}{9k}$ (D) $\sum_{n=1}^{\infty} \frac{1}{1+9k^2}$
- (B) $\sum_{n=1}^{\infty} \frac{1}{k+9}$ (E) $\sum_{n=1}^{\infty} \frac{k}{1+9k^2}$
- (C) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{k+9}}$

Which of the following infinite series has increasing terms?

1143. (A) $\sum_{n=1}^{\infty} (n-2^n)$ (D) $\sum_{n=1}^{\infty} \left(\frac{n}{4n-1}\right)$
- (B) $\sum_{n=1}^{\infty} \left(\frac{n^n}{n!}\right)$ (E) $\sum_{n=1}^{\infty} \left(\frac{10^n}{(2n)!}\right)$
- (C) $\sum_{n=1}^{\infty} \left(\frac{n}{1-2n}\right)$

All of the following are examples of a geometric series except

1142. (A) $1+1+1+1+1+\dots$
- (B) $1+2+3+4+5+\dots$
- (C) $1+2+4+6+8+\dots$
- (D) $-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+\dots$
- (E) $\frac{2}{10}+\frac{2}{10^2}+\frac{2}{10^3}+\frac{2}{10^4}+\frac{2}{10^5}+\dots$

2. Taylor Series

1. Taylor Series

Which of the following generate Taylor series?

1090. (A) $f(x) = \frac{1}{1-x}$ about 0
- (B) $f(x) = \ln(x-1)$ about 1
- (C) $f(x) = \sqrt{x-2}$ about 2
- (D) $f(x) = \sqrt{1+x}$ about -1
- (E) $f(x) = \tan x$ about $\frac{\pi}{2}$

For what values of x does the following power series converge?

- $$\sum_{n=1}^{\infty} \frac{x^n}{n}$$
- (A) $-1 < x < 1$ (D) $-1 < x \leq 1$
- (B) $-1 \leq x \leq 1$ (E) $x > 1$ or $x < 1$
- (C) $-1 \leq x < 1$

For what values of x does the following power series diverge?

- $$\sum_{n=1}^{\infty} \frac{(x+3)^n}{n!}$$
- (A) $-3 \leq x < 3$ (D) $-3 < x < 3$
- (B) $-3 \leq x \leq 3$ (E) $\{ \}$
- (C) $-3 < x \leq 3$

What is the coefficient of $(x - \frac{\pi}{3})^2$ in the Taylor series about $\frac{\pi}{3}$ of $f(x) = \sin x$?

1024. (A) $\frac{1}{4}$ (D) $\frac{\sqrt{3}}{4}$
- (B) $-\frac{1}{4}$ (E) $-\frac{\sqrt{3}}{4}$
- (C) $\frac{1}{3\sqrt{2}}$

What is the 3rd order Taylor polynomial at $x = 0$ for $f(x) = \sin x$?

- (A) $x + \frac{x^3}{3!}$ (D) $x - \frac{x^3}{3!}$
- (B) $x - \frac{x^2}{2!} + \frac{x^3}{3!}$ (E) $x + \frac{x^2}{2!} + \frac{x^3}{3!}$
- (C) $x + \frac{x^2}{2!} - \frac{x^3}{3!}$

What is the 2nd order Taylor polynomial at $x = \pi$ for $f(x) = \cos x$?

- (A) $-1 + (x - \pi) - \frac{(x - \pi)^2}{2!}$
- (B) $1 - \frac{(x - \pi)^2}{2!}$
- (C) $-1 + \frac{(x - \pi)^2}{2!}$
- (D) $x - \pi + \frac{(x - \pi)^2}{2!}$
- (E) $1 - (x - \pi) + \frac{(x - \pi)^2}{2!}$

What is an approximation for $\ln(0.7)$ using the first three terms of the Taylor series

1021. $f(x) = \ln(1+x)$ about $x = 0$?
- (A) -0.340 (D) -0.355
- (B) -0.349 (E) -0.357
- (C) **-0.354**