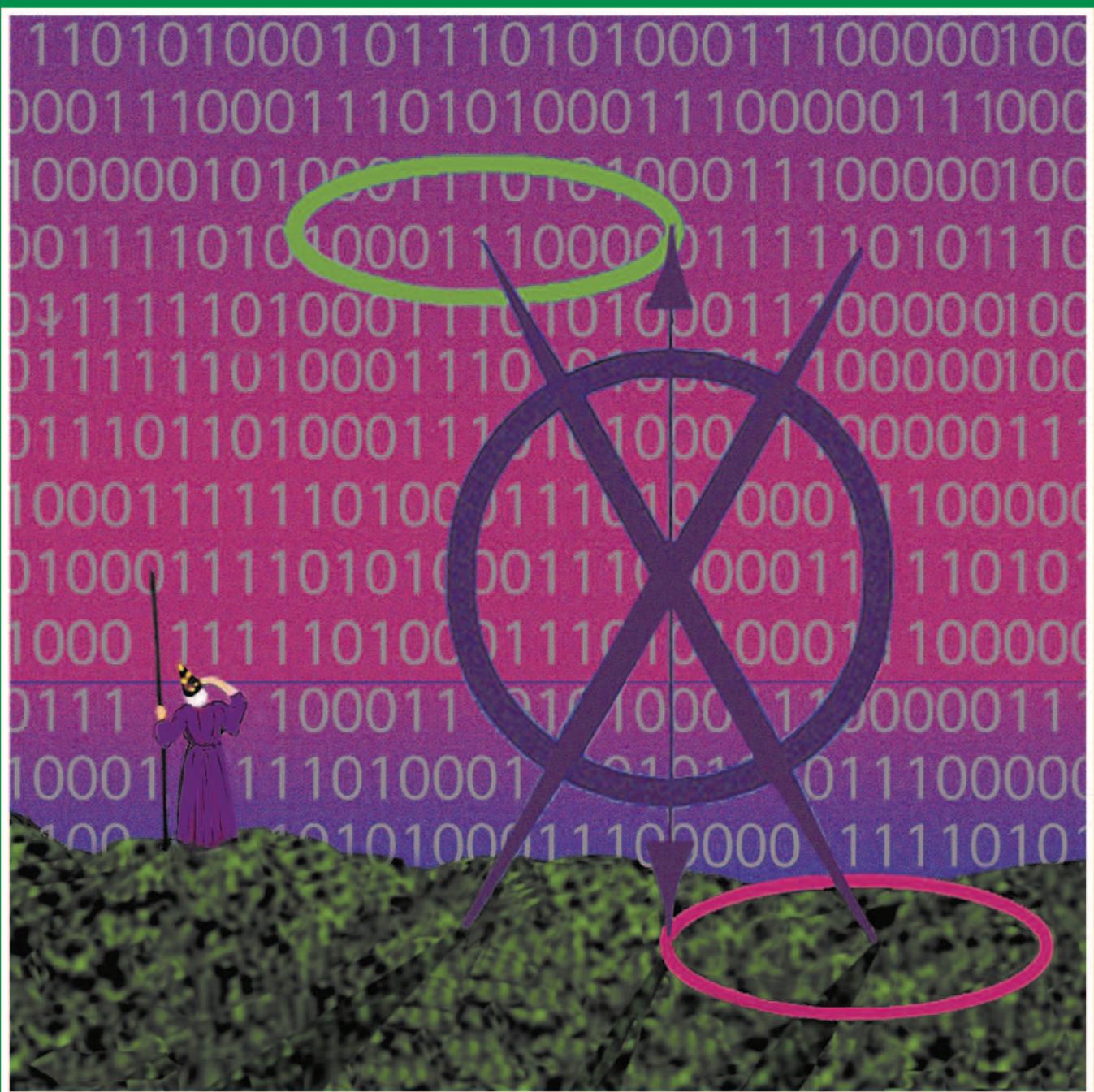


# GEOMETRY & TRIGONOMETRY



QUESTION CATALOGUE

# Geometry

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# I. LOGIC

## A. Logical Equivalence

4999. Given the true statement, "All students attending Learnalot High School are under 20 years of age." Which statement must also be true?

- (A) Jay is 17; therefore Jay is a student at Learnalot.
- (B) Mitch does not attend Learnalot; therefore Mitch is at least 20 years old.
- (C) Dorothy attends high school; therefore Dorothy is under 20 years old.
- (D) Stanley is 23 years old; therefore Stanley is not a student at Learnalot.**

4997. Which statement is logically equivalent to the negation of  $\sim r \wedge s$ ?

- (A)  $r \vee \sim s$**
- (C)  $r \wedge \sim s$
- (B)  $r \vee s$
- (D)  $r \wedge s$

4995. Given the true statement, "If a person drives a car, he or she must be at least 16 years old." Which statement is logically correct?

- (A) David is 16 years old; therefore he drives a car.
- (B) Maria does not drive a car; therefore she is not yet 16 years old.
- (C) Noel is not yet 16 years old; therefore he does not drive a car.**
- (D) Irene drives a car; therefore she is not yet

4993. Which is the negation of the statement, "Chris likes to paint or Rona likes soccer"?

- (A) Chris likes to paint and Rona does not like soccer.
- (B) Chris does not like to paint or Rona does not like soccer.
- (C) Chris does not like to paint and Rona does not like soccer.**
- (D) Chris does not like to paint or Rona likes soccer.

4991. If  $p \rightarrow q$  is a true statement and  $\sim q$  is a true statement, then it follows that

- (A)  $p$  must be a true statement
- (B)  $p$  must be a false statement**
- (C)  $p$  may be either a true or false statement
- (D)  $q$  must be a true statement

4990. Which statement is the negation of  $m \vee \sim n$ ?

- (A)  $\sim m \wedge n$**
- (C)  $\sim m \vee \sim n$
- (B)  $\sim m \vee n$
- (D)  $\sim m \wedge \sim n$

4988. Which statement is logically equivalent to  $\sim a \rightarrow b$ ?

- (A)  $a \rightarrow \sim b$
- (C)  $\sim b \rightarrow a$**
- (B)  $\sim a \rightarrow \sim b$
- (D)  $b \rightarrow \sim a$

4987. Which statement is the negation of  $\sim p \vee q$ ?

- (A)  $p \vee \sim q$
- (C)  $\sim p \vee \sim q$**
- (B)  $p \wedge \sim q$
- (D)  $\sim p \wedge \sim q$

4981. The statement  $\sim(\sim p \wedge \sim q)$  is logically equivalent to

- (A)  $p \wedge q$
- (C)  $\sim p \wedge \sim q$**
- (B)  $p \vee q$
- (D)  $\sim p \vee \sim q$

4980. Which statement is logically equivalent to  $a \rightarrow \sim b$ ?

- (A)  $\sim b \rightarrow \sim a$
- (C)  $b \rightarrow \sim a$**
- (B)  $\sim b \rightarrow a$
- (D)  $b \rightarrow a$

# 1. Logical Operations

## i. Original and Contrapositive

4985. Which is logically equivalent to the statement,

"If I live in New York, then I live in the United States"?

- (A) If I live in the United States, then I live in New York.
- (B) If I do not live in New York, then I do not live in the United States.
- (C) If I do not live in the United States, then I do not live in New York.**
- (D) If I do not live in the United States, then I live in New York.

4978. Which is logically equivalent to the statement, "Students who master their lessons pass their tests"?

- (A) Students who pass their tests master their lessons.
- (B) Students who do not master their lessons do not pass their tests.
- (C) Students who do not pass their tests do not master their lessons.**
- (D) Students who do not pass their tests master their lessons.

4976. Which statement is logically equivalent to the statement, "If the choir does not cut a record, it does not sing well"?

- (A) If the choir cuts a record, it sings well.
- (B) If the choir sings well, it cuts a record.**
- (C) If the choir sings well, it does not cut a record.
- (D) If the choir does not sing well, it does not cut a record.

4969. Which is logically equivalent to the statement, "If Jeff passes math, then he will be happy"?

- (A) Jeff passes math and he is happy.
- (B) If Jeff does not pass math, then he will be happy.
- (C) If Jeff passes math, then he will not be happy.
- (D) If Jeff is not happy, then Jeff did not pass math.**

4961. Which statement is logically equivalent to  $\sim a \rightarrow b$ ?

- (A)  $a \rightarrow \sim b$
- (C)  $b \rightarrow \sim a$**
- (B)  $\sim b \rightarrow a$
- (D)  $\sim b \rightarrow \sim a$

4932. Which statement is logically equivalent to  $\sim p \rightarrow q$ ?

- (A)  $q \rightarrow p$
- (C)  $q \rightarrow \sim p$**
- (B)  $\sim q \rightarrow p$
- (D)  $\sim q \rightarrow \sim p$

4929. Given the true statements, "If George is a dog, then Charlie is a cat," and "Charlie is not a cat." Which statement must also be true?

- (A) Charlie is a cat.
- (C) George is a cat.**
- (B) Charlie is not a dog.
- (D) George is not a dog.**

4927. If  $B \rightarrow \sim C$  and  $C$  are both true statements, then which conclusion must be true?

- (A)  $B$**
- (C)  $\sim C$**
- (B)  $\sim B$
- (D)  $B \rightarrow C$

4924. If  $a \rightarrow b$  and  $b \rightarrow c$  are both true statements, then which statement must also be true?

- (A)  $a \rightarrow \sim c$
- (C)  $c \rightarrow a$**
- (B)  $\sim a \rightarrow c$
- (D)  $\sim c \rightarrow \sim a$**

# I. LOGIC

## B. Translating Logic Statements

4716. Let  $p$  represent " $x > 10$ " and let  $q$  represent " $x$  is a multiple of 5." Which is true if  $x = 26$ ?

- (A)  $p \vee q$  (C)  $p \wedge q$   
 (B)  $p \rightarrow q$  (D)  $p \leftrightarrow q$

4713. Which statement must *always* be true?

- (A)  $p \rightarrow q$  (C)  $\sim p \rightarrow \sim q$   
 (B)  $q \wedge \sim q$  (D)  $p \vee \sim p$

4711. Let  $p$  represent " $x$  is odd" and let  $q$  represent " $x > 15$ ." Which statement is true if  $x = 13$ ?

- (A)  $p \wedge \sim q$  (C)  $p \rightarrow q$   
 (B)  $\sim p \vee q$  (D)  $p \wedge q$

4707. If  $p \wedge \sim q$  is true, which statement must be true?

- (A)  $\sim p$  (C)  $p \vee q$   
 (B)  $q$  (D)  $p \vee q$

4704. Let  $p$  represent " $x > 5$ " and let  $q$  represent " $x$  is a multiple of 3." If  $x = 12$ , which statement is false?

- (A)  $p \vee q$  (C)  $p \rightarrow q$   
 (B)  $\sim q \wedge p$  (D)  $p \leftrightarrow q$

4700. The statement "If  $p$ , then  $q$ " is false if and only if

- (A)  **$p$  is true and  $q$  is false** (C)  $p$  is false and  $q$  is false  
 (B)  $p$  is true and  $q$  is true (D)  $p$  is false and  $q$  is true

4699. If  $p$  represents the statement " $x$  is an integer" and  $q$  represents the statement " $x$  is a prime number," which statement is not true when  $x = 7$ ?

- (A)  $p \rightarrow q$  (C)  $p \wedge \sim q$   
 (B)  $p \leftrightarrow q$  (D)  $p \vee q$

4696. Which value of  $y$  will make the sentence  $(y^2 > 9) \wedge (y < 0)$  true?

- (A) **-4** (C) 3  
 (B) -3 (D) 4

4692. Which statement would be a correct heading for the last column in the table below?

$p$	$q$	?
T	T	F
T	F	F
F	T	T
F	F	F

- (A)  $p \rightarrow q$  (C)  $\sim p \wedge q$   
 (B)  $p \vee q$  (D)  $p \leftrightarrow q$

4689. Let  $p$  represent the statement "All sides are congruent," and let  $q$  represent the statement "All angles are congruent."

The statement  $p \wedge \sim q$  is true for a

- (A) rectangle (C) square  
 (B) rhombus (D) trapezoid

4685. Which statement is true if  $p$  is false and  $q$  is true?

- (A)  $\sim p \rightarrow q$  (C)  $p \vee \sim q$   
 (B)  $q \rightarrow p$  (D)  $p \wedge q$

4678. If  $p \vee q$  is false, then

- (A) both  $p$  and  $q$  are true (C)  $p$  is true and  $q$  is false  
 (B)  $p$  is false and  $q$  is true (D) **both  $p$  and  $q$  are false**

4674. The statement " $x$  is divisible by 3 and  $x$  is greater than 3" is true for which whole number?

- (A) 5 (C) 3  
 (B) 6 (D) 4

# 1. Logical Operations

## iii. Performing Logic Operations

4687. In the accompanying truth table, which statement should be the heading for column 3?

Column 1	Column 2	Column 3
$p$	$q$	?
T	T	T
T	F	F
F	T	F
F	F	F

- (A)  $p \rightarrow q$  (C)  $p \wedge q$   
 (B)  $p \leftrightarrow q$  (D)  $p \vee q$

4675. Given the truth table below, which would be the best heading for the last column?

$p$	$q$	?
T	T	T
T	F	F
F	T	F
F	F	T

- (A)  $p \wedge q$  (C)  $p \vee q$   
 (B)  $p \rightarrow q$  (D)  $p \leftrightarrow q$

4670. Which statement is always true when  $p$  is false?

- (A)  $p \vee q$  (C)  $p \wedge q$   
 (B)  $p \leftrightarrow q$  (D)  $\sim p$

4663. Let  $p$  represent the statement " $x + 3$  is even" and let  $q$  represent the statement " $x$  is prime." If the replacement set for  $x$  is 1,2,3,4, find the value of  $x$  for which the statement  $p \wedge q$  is true.

**3**

4661. If  $p$  represents "x is prime" and  $q$  represents " $2x = 10$ ," then which statement is true when  $x = 3$ ?

- (A)  $p \vee q$  (C)  $p \rightarrow q$   
 (B)  $p \wedge q$  (D)  $q$

4653. If  $p$  is true and  $q$  is false, which statement must be false?

- (A)  $p \vee q$  (C)  $p \rightarrow q$   
 (B)  $q \rightarrow p$  (D)  $p \wedge \sim q$

4652. Let  $p$  represent the statement " $x \geq 5$ " and let  $q$  represent the statement " $2x = 4$ ." Which is true if  $x = 6$ ?

- (A)  $p \wedge q$  (C)  $p \rightarrow q$   
 (B)  $p \vee q$  (D)  $p \leftrightarrow q$

4647. If  $p \rightarrow q$  is false, then

- (A) both  $p$  and  $q$  are true (C) both  $p$  and  $q$  are false  
 (B)  **$p$  is true and  $q$  is false** (D)  $p$  is false and  $q$  is true

4644. Which of the following is true when  $p$  is false and  $q$  is false?

- (A)  $p \vee q$  (C)  $p \leftrightarrow q$   
 (B)  $p \wedge q$  (D)  $\sim(\sim p)$

4642. Given the true statement: "If John lives in Buffalo, then he lives in New York State." If John does not live in New York State, which statement is a logical conclusion?

- (A) **He does not live in Buffalo.**  
 (B) He might still live in Buffalo.  
 (C) He lives in Chicago.  
 (D) He does not live in the United States.

5010. Given:

Bill vacations in Canada or the United States.  
If the metric system is not used, then gasoline is not sold in liters.  
If Bill vacations in Canada, then gasoline is sold in liters.  
Bill does not vacation in the United States.

Let  $C$  represent: "Bill vacations in Canada."

Let  $S$  represent: "Bill vacations in the United States."

Let  $M$  represent: "The metric system is used."

Let  $L$  represent: "Gasoline is sold in liters."

Prove: The metric system is used.

4996. Given: Either I go to summer school or I go on vacation.

If I go on vacation, then I will swim every day.

If I swim every day, then I will try out for the team.

I did not try out for the team.

Let  $S$  represent: "I go to summer school."

Let  $G$  represent: "I go on vacation."

Let  $E$  represent: "I swim every day."

Let  $T$  represent: "I try out for the team."

Prove: I go to summer school.

4992. Given: Either Evan or Rona went out on Saturday night.

If Evan went out on Saturday night, then he studied on Tuesday.

If Evan studied on Tuesday, then Anita did not do her schoolwork on Sunday.  
Anita did her schoolwork on Sunday.

Let  $E$  represent: "Evan went out on Saturday night."

Let  $R$  represent: "Rona went out on Saturday night."

Let  $T$  represent: "Evan studied on Tuesday."

Let  $A$  represent: "Anita did her schoolwork on Sunday."

Prove: Rona went out on Saturday night.

4986. Given:

Marilyn receives good grades in English.

If Marilyn likes social studies, then she doesn't like computers.

If Marilyn receives good grades in English and doesn't like geometry, then she likes computers.

Marilyn likes social studies.

Let  $E$  represent: "Marilyn receives good grades in English."

Let  $S$  represent: "Marilyn dislikes social studies."

Let  $C$  represent: "She likes computers."

Let  $G$  represent: "Marilyn likes geometry."

Prove: Marilyn likes geometry.

4983. Given:

If Lori washes dishes, then Keri dries them.

Lori washes dishes or Jason mows the lawn.

We get an allowance if Jason mows the lawn.

If we get an allowance, we can see a movie.

Keri doesn't dry the dishes.

Let  $L$  represent: "Lori washes dishes."

Let  $K$  represent: "Keri dries the dishes."

Let  $J$  represent: "Jason mows the lawn."

Let  $A$  represent: "We get an allowance."

Let  $M$  represent: "We can see a movie."

Using  $L$ ,  $K$ ,  $J$ ,  $A$ , and  $M$ , prove: "We can see a movie."

4979. Given: If laws are good and strictly enforced, then crime will diminish. If laws are not strictly enforced, then the problem is critical. Crime has not diminished. Laws are good.

Let  $G$  represent: "Laws are good."

Let  $S$  represent: "Laws are strictly enforced."

Let  $D$  represent: "Crime has diminished."

Let  $P$  represent: "The problem is critical."

Using  $G$ ,  $S$ ,  $D$ , and  $P$ , prove: "The problem is critical."

4972. Given:

Jason is smart.

If Jason is smart and has a job, then he will earn a good income.

If Jason works hard, then he will have a job.

If Jason does not work hard, then he will be unhappy.

Jason does not have a good income.

Let  $S$  represent: "Jason is smart."

Let  $W$  represent: "Jason works hard."

Let  $J$  represent: "He has a job."

Let  $I$  represent: "He will earn a good income."

Let  $U$  represent: "He will be unhappy."

Prove: Jason will be unhappy.

4963. Given:

If the radio is on and the television works, then there is electricity.

If the lights go out, then there is no electricity.

The lights go out.

The radio is on.

Let  $R$  represent: "The radio is on."

Let  $T$  represent: "The television works."

Let  $E$  represent: "There is electricity."

Let  $L$  represent: "The lights go out."

Using  $R$ ,  $T$ ,  $E$ , and  $L$ , prove: "The television does not work."

## II. GEOMETRY

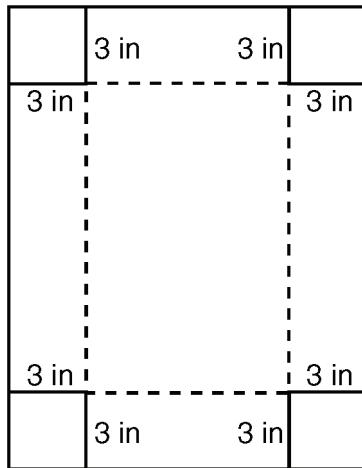
### A. Area, Perimeter & Volume of Polygons

4235. A rectangular piece of cardboard is to be formed into an uncovered box. The piece of cardboard is 2 centimeters longer than it is wide. A square that measures 3 centimeters on a side is cut from each corner. When the sides are turned up to form the box, its volume is 765 cubic centimeters. Find the dimensions, in centimeters, of the original piece of cardboard.

**21 by 23**

4190. A garden is 20 meter long and 15 meter wide. What is the distance between a plant at the southwest corner and a plant at the northeast corner?  
 (A) 18 m    (C) 25 m  
 (B) 20 m    (D) 26 m

4047. Deborah built a box by cutting 3-inch squares from the corners of a rectangular sheet of cardboard, as shown in the accompanying diagram, and then folding the sides up. The volume of the box is 150 cubic inches, and the longer side of the box is 5 inches more than the shorter side. Find the number of inches in the shorter side of the *original* sheet of cardboard.



**11, and appropriate work is shown, such as solving the quadratic equation  $3x(x + 5) = 150$  or trial and error with at least three trials and appropriate checks.**

4045. Tina's preschool has a set of cardboard building blocks, each of which measures 9 inches by 9 inches by 4 inches. How many of these blocks will Tina need to build a wall 4 inches thick, 3 feet high, and 12 feet long?

**64, and appropriate work is shown, such as calculating  $\frac{(36 \times 144)}{(9 \times 9)}$  or drawing a labeled diagram.**

3952. Javon's homework is to determine the dimensions of his rectangular backyard. He knows that the length is 10 feet more than the width, and the total area is 144 square feet. Write an equation that Javon could use to solve this problem. Then find the dimensions, in feet, of his backyard.

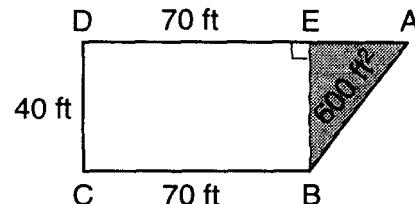
$$x(x + 10) = 144$$

3523. If one side of a regular hexagon is represented by  $3x + 5$ , the perimeter of the hexagon can be represented by  
 (A)  $3x + 30$     (C)  $18x + 30$   
 (B)  $18x + 5$     (D)  $18x - 30$

## 1. Geometry of Measurement

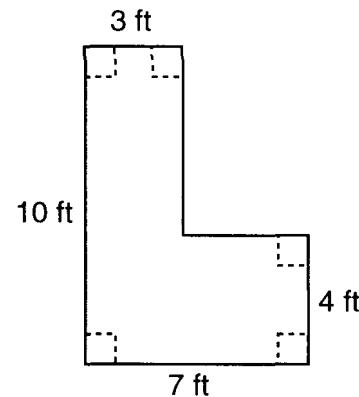
### ii. Word Problems for Area, Perimeter & Volume

3705. The plan of a parcel of land is represented by trapezoid  $ABCD$  in the accompanying diagram. If the area of  $\triangle ABE$  is 600 square feet, find the minimum number of feet of fence needed to completely enclose the entire parcel of land,  $ABCD$ .



**260**

3704. Keesha wants to tile the floor shown in the accompanying diagram. If each tile measures 1 foot by 1 foot and costs \$2.99, what will be the total cost, including an 8% sales tax, for tiling the floor?



**\$148.54**

3698. If the length of a rectangular prism is doubled, its width is tripled, and its height remains the same, what is the volume of the new rectangular prism?

- (A) double the original volume  
 (B) triple the original volume  
**(C) six times the original volume**  
 (D) nine times the original volume

1352. The Pentagon building in Washington, D.C., is shaped like a regular pentagon. If the length of one side of the Pentagon is represented by  $n + 2$ , its perimeter would be represented by

- (A)  $5n + 10$**     (C)  $n + 10$   
 (B)  $5n + 2$     (D)  $10n$

1190. An 8 by 10 inch photo has a frame of uniform width placed around it.

a If the uniform width of the frame is  $x$  inches, express the outside dimensions of the picture frame in terms of  $x$ .

b If the area of the picture and frame is  $143 \text{ in}^2$ , what is the uniform width of the frame?

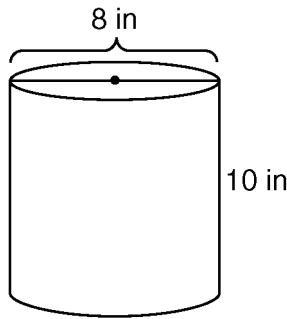
**II. GEOMETRY****A. Area, Perimeter & Volume of Polygons**

5253. Tim has a rectangular prism with a length of 10 centimeters, a width of 2 centimeters, and an unknown height. He needs to build another rectangular prism with a length of 5 centimeters and the same height as the original prism. The volume of the two prisms will be the same. Find the width, in centimeters, of the new prism.

4

5159. A right circular cylinder has a volume of 1,000 cubic inches and a height of 8 inches. What is the radius of the cylinder to the *nearest tenth of an inch*?  
 (A) 6.3                            (C) 19.8  
 (B) 11.2                            (D) 39.8

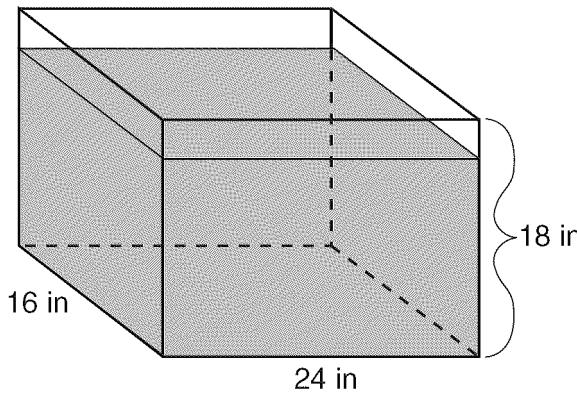
4259. A storage container in the shape of a right circular cylinder is shown in the accompanying diagram.



What is the volume of this container, to the *nearest hundredth*?

- (A)  $56.55 \text{ in}^3$                             (C)  $251.33 \text{ in}^3$   
 (B)  $125.66 \text{ in}^3$                             (D) **502.65 in<sup>3</sup>**

4251.



As shown in the accompanying diagram, the length, width, and height of Richard's fish tank are 24 inches, 16 inches, and 18 inches, respectively. Richard is filling his fish tank with water from a hose at the rate of 500 cubic inches per minute. How long will it take, to the *nearest minute*, to fill the tank to a depth of 15 inches?

**12 minutes**

4066. A box in the shape of a cube has a volume of 64 cubic inches. What is the length of a side of the box?

- (A)  $21\frac{1}{3}$  in                                    (C) 8 in  
 (B) 16 in    (D) **4 in**

**1. Geometry of Measurement****iii. Using Formula for Volume & Surface Area of Prisms**

1348. The dimensions of a brick, in inches, are 2 by 4 by 8. How many such bricks are needed to have a total volume of exactly 1 cubic foot?

27

537. The dimensions of a brick, in inches, are 2 by 4 by 8. How many such bricks are needed to have a total volume of exactly 1 cubic foot?

27

446. The volume of a rectangular solid is 24 cubic centimeters. If the width is 2 centimeters and the length is 3 centimeters, what is the height, in *centimeters*, of the solid?

4

434. If the length of the edge of a cube is  $5x$ , the volume of the cube is

- (A)  $5x^3$     (C)  $125x^3$   
 (B)  $15x^3$     (D)  $625x^3$

393. A side of a cube measures 4 centimeters and a side of a smaller cube measures 2 centimeters. The volume of the larger cube is how many times the volume of the smaller cube?

- (A) 6    (C) 8  
 (B) 2    (D) 4

338. The length of a rectangular solid is 3.0 meters, the width is 0.6 meter, and the height is 0.4 meter. Find, to the nearest tenth, the number of cubic meters in the volume of the solid.

0.7

299. If the volume of a cube is 64 cubic centimeters, how many centimeters are in the length of an edge of the cube?

4

292. The volume of a rectangular solid is 180 cubic centimeters. The length is 10 centimeters and the width is 4 centimeters. Using the formula  $V = lwh$ , find the number of centimeters in the height.

4.5

286. If the edge of a cube is doubled, the volume is multiplied by  
 (A) 6    (C) 3  
 (B) 2    (D) 8

233. If  $V = lwh$ , what is the value of  $V$  when  $l = 2$ ,  $w = 3$ , and  $h = 4x$ ?

- (A)  $9x$     (C)  $5 + 4x$   
 (B) **24x**    (D)  $6 + 4x$

224. The length of an edge of a cube is 2 inches. How many cubic inches are there in the volume of the cube?

8

182. What is the volume, in cubic centimeters, of a cube whose edge measures 2 centimeters?

8

150. Express, in cubic feet, the volume of a room whose dimensions are 12 feet long by 10 feet wide by 8 feet high.

**960**

128. The length of an edge of a cube is represented by  $5x$ . Which expression represents the volume of the cube?

- (A)  $10x^2$     (C)  $5x^3$   
 (B)  $25x^2$     (D) **125x<sup>3</sup>**

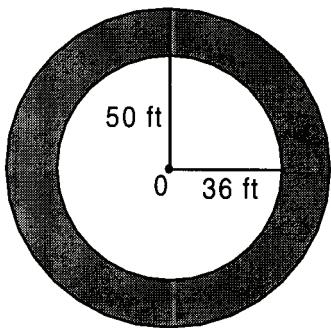
98. The length of the edge of a cube is represented by  $e$ . Express the volume of the cube in terms of  $e$ .

 $e^3$

## II. GEOMETRY

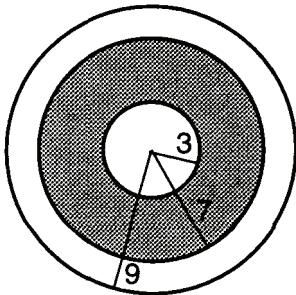
### C. Shaded Region Problems

549. If asphalt pavement costs \$0.78 per square foot, determine, to the *nearest cent*, the cost of paving the shaded circular road with center  $O$ , an outside radius of 50 feet, and an inner radius of 36 feet, as shown in the accompanying diagram.



**\$2,950.33 and a correct method is shown, such as area  $1204\pi$  square feet multiplied by \$0.78.**

539. A target shown in the accompanying diagram consists of three circles with the same center. The radii of the circles have lengths of 3 inches, 7 inches, and 9 inches.



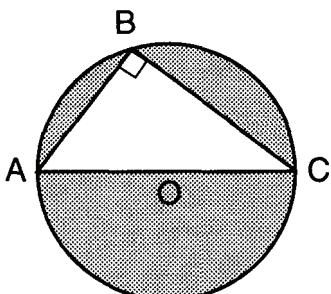
- a What is the area of the shaded region to the *nearest tenth* of a square inch?

- b To the *nearest percent*, what percent of the target is shaded?

**a 125.6 or 125.7**

**b 49**

495. In the accompanying diagram, right triangle ABC with the right angle at B is inscribed in circle O, AC is a diameter,  $BC = 12$  centimeters, and  $AB = 9$  centimeters. Find the area of the shaded region to the *nearest square centimeter*.

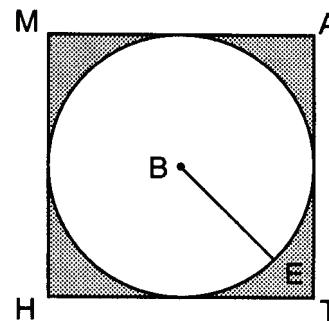


**123**

## 1. Geometry of Measurement

### ii. Shaded Region Problems (Ext Task)

532. In the accompanying diagram, circle B is inscribed in square MATH and radius BE = 5.



a Find the length of  $\overline{MA}$ .

b Find the area of square MATH.

c Find the length of diagonal  $\overline{HA}$  in simplest radical form.

d Find the area of the shaded region in terms of  $\pi$ .

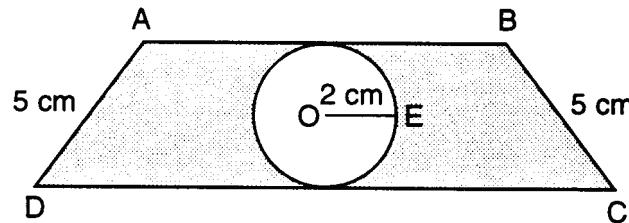
**a 10**

**b 100**

**c  $10\sqrt{2}$**

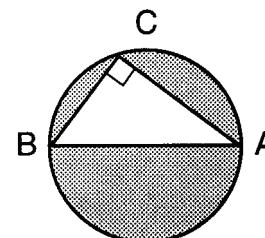
**d  $100 - 25\pi$**

523. In the accompanying diagram, the length of each leg of isosceles trapezoid ABCD is 5 centimeters. The length of  $\overline{DC}$  is 6 centimeters longer than the length of  $\overline{AB}$ , and the perimeter of trapezoid ABCD is 36 centimeters. Circle O is inscribed in the trapezoid. Radius OE equals 2 centimeters. Find the area of the shaded region to the *nearest tenth* of a square centimeter.



**39.4**

479. In the accompanying diagram, right triangle ABC is inscribed in a circle, BA is a diameter,  $BC = 6$  centimeters, and  $AC = 8$  centimeters. Find the area of the shaded portion to the *nearest tenth* of a square centimeter. [Use  $\pi = 3.14$ .]

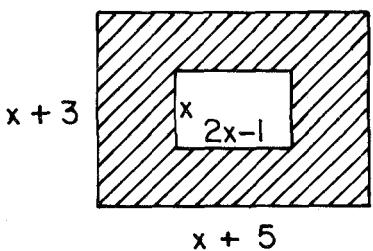


**54.5**

## II. GEOMETRY

### C. Shaded Region Problems

255. In the accompanying diagram, the width of the inner rectangle is represented by  $x$  and the length by  $2x - 1$ . The width of the outer rectangle is represented by  $x + 3$  and the length by  $x + 5$ .



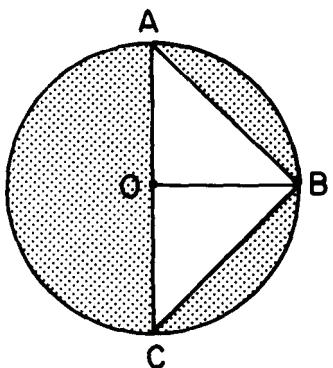
- a Express the area of  
 (1) the inner rectangle as a binomial in terms of  $x$   
 (2) the outer rectangle as a trinomial in terms of  $x$   
 (3) the shaded region as a trinomial in terms of  $x$

b If the perimeter of the outer rectangle is 24, what is the value of  $x$ ?

- a (1)  $2x^2 - x$   
 (2)  $x^2 + 8x + 15$   
 (3)  $-x^2 + 9x + 15$

b 2

218. In the accompanying figure,  $\triangle ABC$  is inscribed in circle  $O$ ,  $\overline{AC}$  is a diameter of circle  $O$ ,  $AC = 8$ , and  $\overline{AB} \cong \overline{BC}$ .



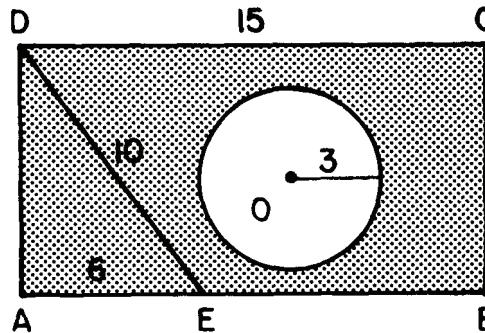
- a Find the measure of minor arc  $AB$ .  
 b Find the measure of  $\angle BOC$ .  
 c Find the measure of  $\angle ABC$ .  
 d Find the area of  $\triangle ABC$ .  
 e Find the area of the shaded region. [Answer may be left in terms of  $\pi$ .]

- a 90  
 b 90  
 c 90  
 d 16  
 e  $16\pi - 16$

## 1. Geometry of Measurement

### ii. Shaded Region Problems (Ext Task)

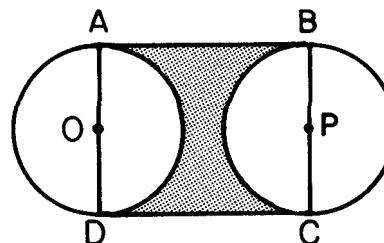
203. In the accompanying diagram,  $ABCD$  is a rectangle,  $E$  is a point on  $\overline{AB}$ ,  $DE = 10$ ,  $AE = 6$ , and  $DC = 15$ . Circle  $O$  has a radius of 3.



- a Find  $AD$ .  
 b Find the area of  $\triangle ADE$ .  
 c Find the area of circle  $O$ .  
 d Find the area of trapezoid  $EBCD$ .  
 e Find the area of the shaded portion.

- a 8  
 b 24  
 c  $9\pi$   
 d 96  
 e  $120 - 9\pi$

195. In the accompanying diagram, circles  $O$  and  $P$  have diameters  $\overline{AD}$  and  $\overline{BC}$ , respectively,  $\overline{AD} \cong \overline{BC}$ ,  $AD = 12$ ,  $ABCD$  is a rectangle, and side  $AB = 15$ . [Answers may be left in terms of  $\pi$ .]



- a What is the perimeter of rectangle  $ABCD$ ?  
 b What is the area of rectangle  $ABCD$ ?  
 c What is the length of the radius of circle  $O$ ?  
 d What is the area of circle  $O$ ?  
 e What is the area of the shaded region in the diagram?

- a 54  
 b 180  
 c 6  
 d  $36\pi$   
 e  $180 - 36\pi$

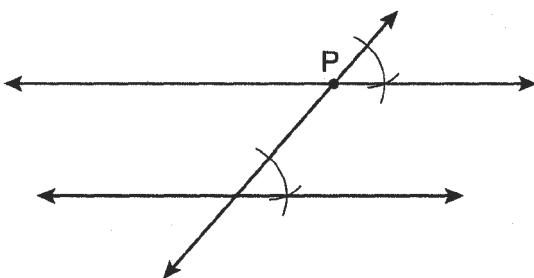
## II. GEOMETRY

### B. Special Angles

## 2. Geometry Relations

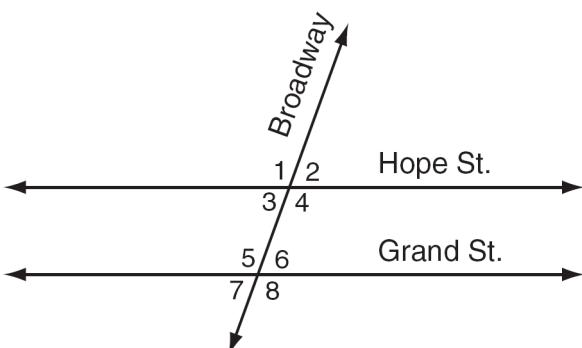
### iv. Angle Relationships with Parallel Lines

5232. Which geometric principle is used to justify the construction below?



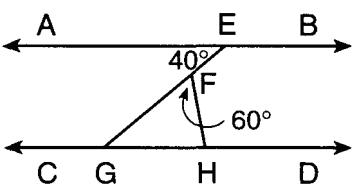
- (A) A line perpendicular to one of two parallel lines is perpendicular to the other.
- (B) Two lines are perpendicular if they intersect to form congruent adjacent angles.
- (C) When two lines are intersected by a transversal and alternate interior angles are congruent, the lines are parallel.
- (D) When two lines are intersected by a transversal and the corresponding angles are congruent, the lines are parallel.**

4388. The accompanying diagram shows two parallel roads, Hope Street and Grand Street, crossed by a transversal road, Broadway.



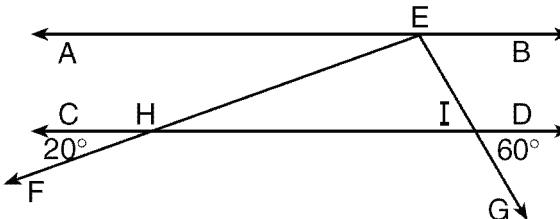
If  $m\angle 1 = 110$ , what is the measure of  $\angle 7$ ?

- (A)  $40^\circ$
  - (C)  $110^\circ$
  - (B)  $70^\circ$**
  - (D)  $180^\circ$
3540. In the accompanying diagram,  $\overleftrightarrow{AEB} \parallel \overleftrightarrow{CGHD}$ ,  $\overleftrightarrow{EFG}$  and  $\overleftrightarrow{FH}$  are drawn,  $m\angle AEF = 40$ , and  $m\angle GFH = 60$ . Find  $m\angle FHD$ .



100

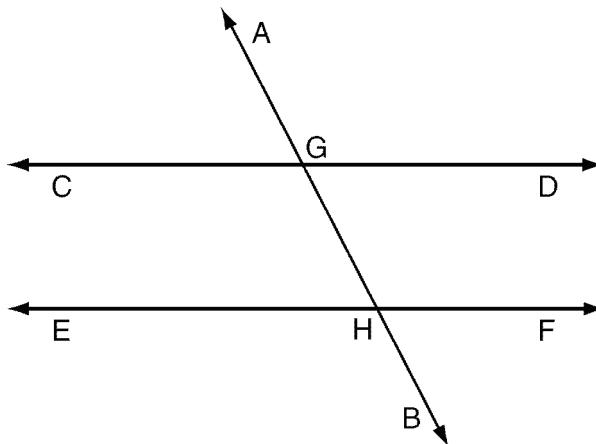
4358. In the accompanying diagram,  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ . From point E on  $\overleftrightarrow{AB}$ , transversals  $\overleftrightarrow{EF}$  and  $\overleftrightarrow{EG}$  are drawn, intersecting  $\overleftrightarrow{CD}$  at H and I, respectively.



If  $m\angle CHF$  and  $m\angle DIG = 60$ , what is  $m\angle HEI$ ?

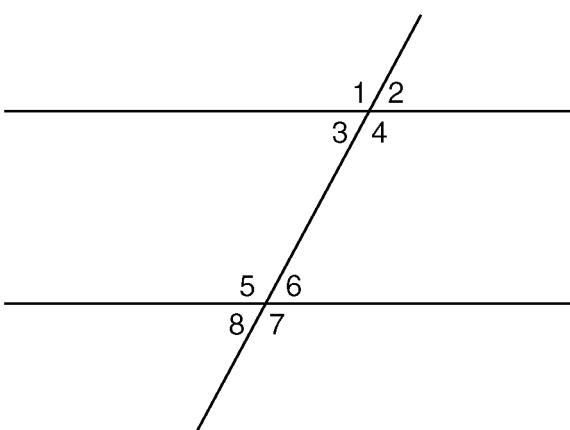
- (A) 60
- (C) **100**
- (B) 80
- (D) 120

4338. In the accompanying diagram  $\overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$ ,  $\overleftrightarrow{AB}$  is a transversal,  $m\angle DGH = 2x$ , and  $m\angle FHB = 5x - 51$ . Find the measure, in degrees, of  $\angle BHE$ .



146

3972. In the accompanying figure, what is one pair of alternate interior angles?



- (A)  $\angle 1$  and  $\angle 2$
- (C)  $\angle 4$  and  $\angle 6$
- (B)  $\angle 4$  and  $\angle 5$**
- (D)  $\angle 6$  and  $\angle 8$

## II. GEOMETRY

## A. Triangle Relationships

975. In  $\triangle ABC$ ,  $m\angle B$  is  $10^\circ$  larger than  $m\angle A$ , and  $m\angle C$  is  $5^\circ$  less than 3 times  $m\angle A$ . Find  $m\angle A$ .

**35**

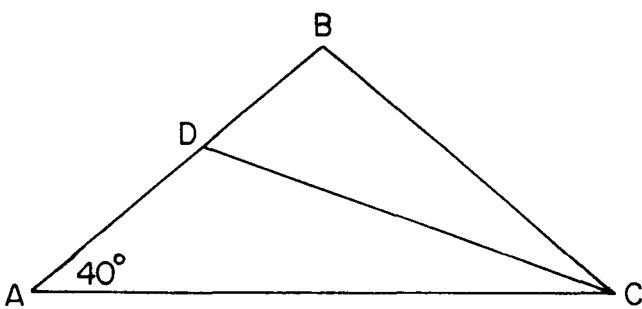
969. Which set of numbers can *not* represent the measures of the angles of a triangle?

- (A) {35, 125, 10}      (C) {30, 110, 40}  
 (B) {20, 100, 60}      (D) {97, 59, 24}

945. If the measures of the angles of a triangle are represented by  $x + 30$ ,  $4x + 30$ , and  $10x - 30$ , the triangle must be

- (A) isosceles      (C) right  
 (B) obtuse      (D) scalene

929. In the accompanying figure,  $\overline{AB} \cong \overline{BC}$ ,  $m\angle A = 40$ , and  $\overline{CD}$  bisects  $\angle ACB$ . Find  $m\angle CDB$ .

**60**

926. If the measures of the angles of a triangle are represented by  $x$ ,  $3x + 6$ , and  $2x - 6$ , find the value of  $x$ .

**30**

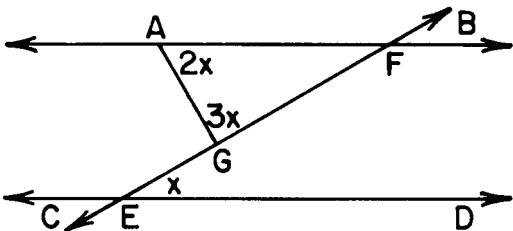
916. In  $\triangle ABC$ ,  $m\angle A:m\angle B:m\angle C = 2:3:4$ . What is  $m\angle C$ ?

**80**

886. In right triangle  $ABC$ , altitude  $\overline{CD}$  is drawn to hypotenuse  $\overline{AB}$ . If  $m\angle A = 40$ , find  $m\angle BCD$ .

**40**

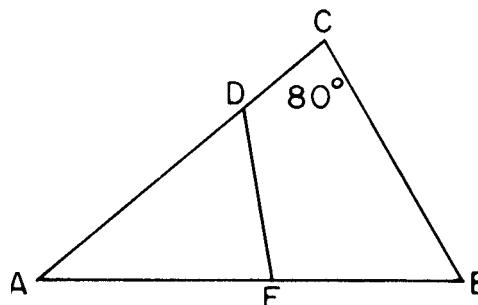
881. In the accompanying figure,  $\overleftrightarrow{EGF}$  intersects  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{CD}$ , and  $\overleftrightarrow{AG}$  is drawn. If  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ ,  $m\angle FED = x$ ,  $m\angle GAF = 2x$ , and  $m\angle FGA = 3x$ , find  $x$ .

**30**

863. In triangle  $ABC$ ,  $m\angle A = 80$  and  $AB > AC$ . What is the smallest angle of triangle  $ABC$ ?

**B or ABC**

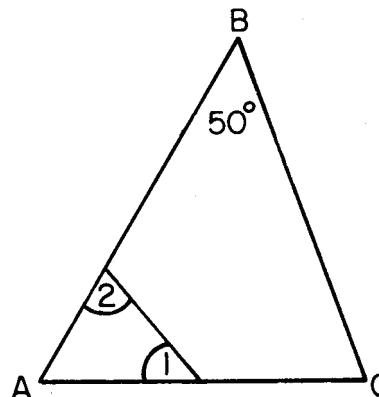
873. In the accompanying diagram of  $\triangle ABC$ ,  $m\angle B = m\angle ADE$  and  $m\angle C = 80$ . Find  $m\angle AED$ .

**80**

858. If two angles of one triangle are congruent to two angles of another triangle, then those triangles *must be* (A) scalene      (C) congruent  
 (B) isosceles      (D) similar

844. Two isosceles triangles with congruent vertex angles *must be* (A) congruent      (C) right  
 (B) equilateral      (D) similar

823. In the accompanying diagram, if  $m\angle 1 = 50$ ,  $m\angle 2 = 70$ , and  $m\angle B = 50$ , find  $m\angle C$ .

**70**

806. If the measures of the three angles of a triangle are represented by  $x$ ,  $2x - 20$ , and  $3x - 10$ , then the triangle is (A) right      (C) isosceles  
 (B) obtuse      (D) equilateral

799. The measures of the angles of a triangle are in the ratio of 1:3:5. Find the number of degrees in the measure of the smallest angle of the triangle.

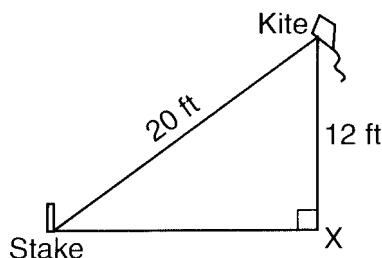
**20**

781. In  $\triangle ABC$ ,  $m\angle A = 70$  and  $m\angle C = 50$ . If  $D$  is a point on  $\overline{AB}$  such that  $\overline{CD}$  bisects  $\angle ACB$ , find  $m\angle CDB$ .

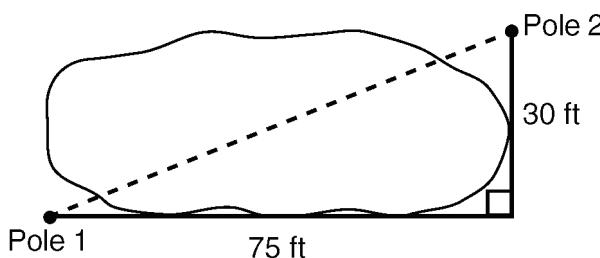
**95**

**II. GEOMETRY****B. Right Triangles**

4271. The accompanying diagram shows a kite that has been secured to a stake in the ground with a 20-foot string. The kite is located 12 feet from the ground, directly over point X. What is the distance, in feet, between the stake and point X?

**16**

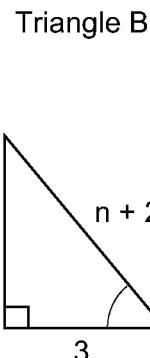
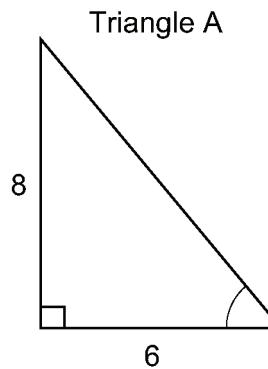
4241. The NuFone Communications Company must run a telephone line between two poles at opposite ends of a lake, as shown in the accompanying diagram. The length and width of the lake are 75 feet and 30 feet, respectively.



What is the distance between the two poles, to the *nearest foot*?

- (A) 105                                    (C) 69  
**(B) 81**                                    (D) 45

3964. In the accompanying diagram, triangle A is similar to triangle B.



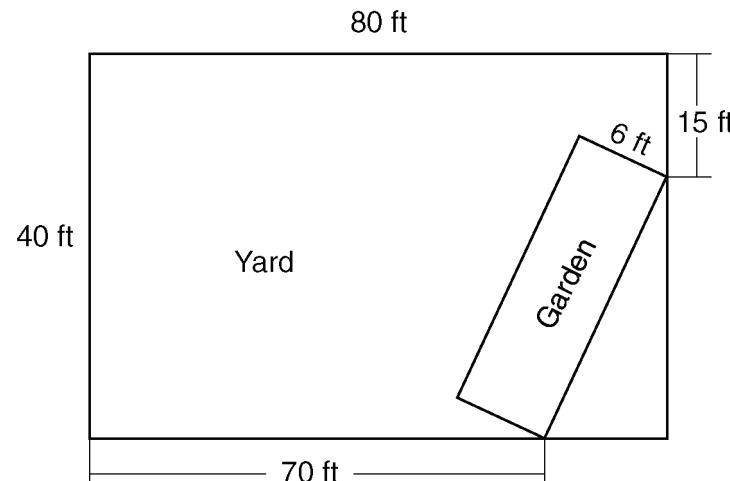
Find the value of  $n$ .

**3**

1260. The diagonals of a rhombus are 8 and 10. Find the measure of a side of the rhombus to the *nearest tenth*.

**6.4**

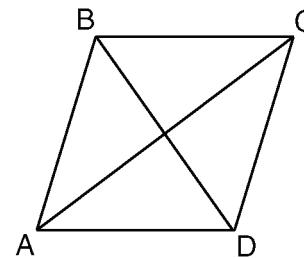
3974. A rectangular garden is going to be planted in a person's rectangular backyard, as shown in the accompanying diagram. Some dimensions of the backyard and the width of the garden are given. Find the area of the garden to the *nearest square foot*.

**162**

3842. In a right triangle, one leg is 3 more than the other, and the hypotenuse is 3 less than twice the shorter leg. Find the numerical value of the perimeter of this triangle.

**36**

3818. In the accompanying diagram of rhombus ABCD, diagonal  $AC = 80$  and  $m\angle BAC = 36$ .



a Find the length of a side of the rhombus to the *nearest tenth*.

b Using your answer in part a, find the perimeter of rhombus ABCD.

c Find the length of diagonal BD to the *nearest integer*.

**a 49.4****b 197.6****c 58**

760. In rectangle ABCD,  $BC = 12$  and  $CD = 16$ . Find  $BD$ .

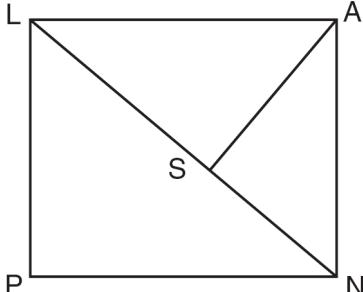
**20**

## II. GEOMETRY

## B. Right Triangles

5271. In a right triangle where one of the angles measures  $30^\circ$ , what is the ratio of the length of the side opposite the  $30^\circ$  angle to the length of the side opposite the  $90^\circ$  angle?  
 (A)  $1:\sqrt{2}$       (C)  $1:3$   
 (B)  $1:2$       (D)  $1:\sqrt{3}$

4517. The accompanying diagram shows part of the architectural plans for a structural support of a building.  $PLAN$  is a rectangle and  $\overline{AS} \perp \overline{LN}$ .



Which equation can be used to find the length of  $\overline{AS}$ ?

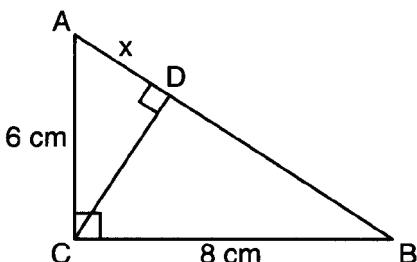
$$\frac{LS}{AS} = \frac{AS}{SN} \quad \text{(A)}$$

$$\frac{AS}{SN} = \frac{AS}{LS} \quad \text{(C)}$$

$$\frac{AN}{LN} = \frac{AS}{LS} \quad \text{(B)}$$

$$\frac{AS}{LS} = \frac{LS}{SN} \quad \text{(D)}$$

4470. In the diagram below, the length of the legs  $\overline{AC}$  and  $\overline{BC}$  of right triangle  $ABC$  are 6 cm and 8 cm, respectively. Altitude  $\overline{CD}$  is drawn to the hypotenuse of  $\triangle ABC$ .



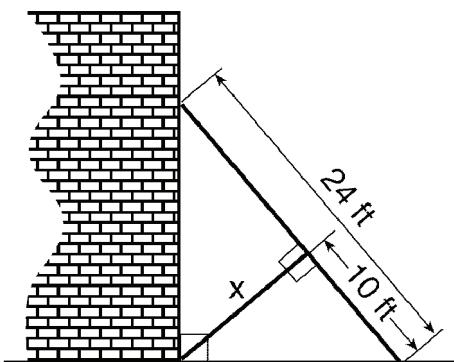
What is the length of  $\overline{AD}$  to the nearest tenth of a centimeter?

- (A) 3.6      (C) 6.4  
 (B) 6.0      (D) 4.0

1299. In right triangle  $RST$ ,  $\overline{TP}$  is the altitude to hypotenuse  $\overline{RS}$ . If  $RP = 11$  and  $SP = 21$ , find the length of  $TP$  to the nearest tenth.

15.2

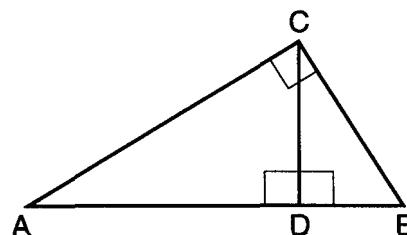
4348. The accompanying diagram shows a 24-foot ladder leaning against a building. A steel brace extends from the ladder to the point where the building meets the ground. The brace forms a right angle with the ladder.



If the steel brace is connected to the ladder at a point that is 10 feet from the foot of the ladder, which equation can be used to find the length,  $x$ , of the steel brace?

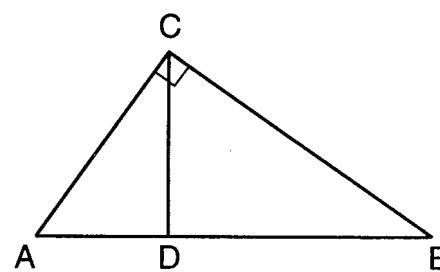
- (A)  $\frac{10}{x} = \frac{x}{14}$       (C)  $10^2 + x^2 = 14^2$   
 (B)  $\frac{10}{x} = \frac{x}{24}$       (D)  $10^2 + x^2 = 24^2$

1325. In the accompanying diagram of right triangle  $ABC$ ,  $\overline{CD}$  is the altitude to hypotenuse  $\overline{AB}$ ,  $CD = 6$ , and  $DB = 4$ . Find the length of  $\overline{AD}$ .



9

1318. In the accompanying diagram of right triangle  $ABC$ , altitude  $\overline{CD}$  is drawn to hypotenuse  $\overline{AB}$ .  $CA = 6$ , and  $AB$  is 7 more than  $AD$ .



a Find  $AD$  to the nearest hundredth.

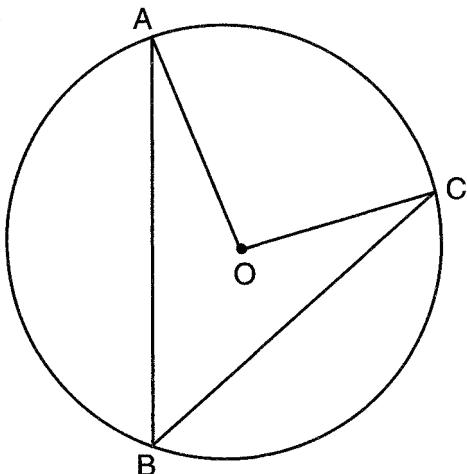
b Using the results from part a, find the length of altitude  $\overline{CD}$  to the nearest tenth.

- a 3.45  
 b 4.9

## II. GEOMETRY

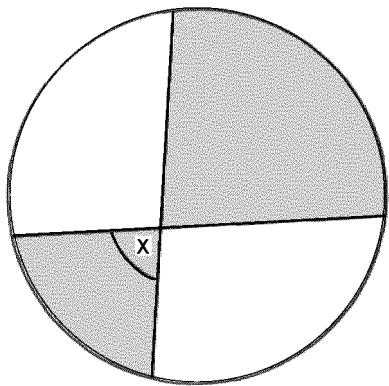
### A. Degrees in a Circle

4430. In the accompanying diagram of circle  $O$ ,  $\overline{AB}$  and  $\overline{BC}$  are chords and  $m\angle AOC = 96$ . What is  $m\angle ABC$ ?



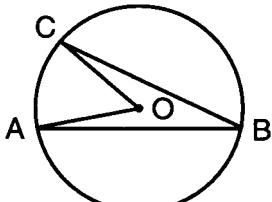
- (A) 32                                  (C) 96  
**(B) 48**                                  (D) 192

4225. The accompanying diagram shows a child's spin toy that is constructed from two chords intersecting in a circle. The curved edge of the larger shaded section is one-quarter of the circumference of the circle, and the curved edge of the smaller shaded section is one-fifth of the circumference of the circle.



- What is the measure of angle  $x$ ?  
(A)  $40^\circ$                               (C)  $81^\circ$   
(B)  $72^\circ$                               (D)  $108^\circ$

2270. In the accompanying figure of circle  $O$ ,  $m\angle AOC = 52$ . Find  $m\angle ABC$ .

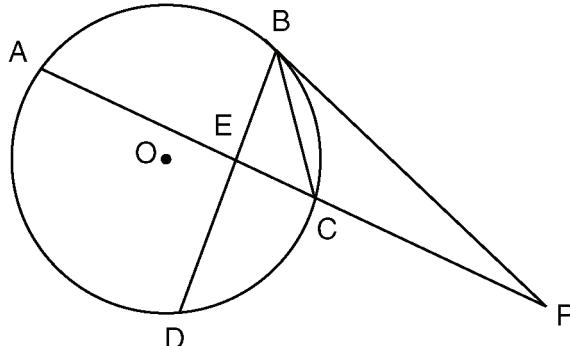


26

## 5. Geometry of the Circle

### i. Angles of the Circle

3938. In the accompanying diagram of circle  $O$ , tangent  $\overline{PB}$ , secant  $\overline{AECP}$ , chord  $\overline{DEB}$ , and chord  $\overline{CB}$  are drawn;  $m\widehat{DC} = 90$ ;  $m\angle DEC = 85$ ;  $BP = 15$ ; and  $CB = 8$ .

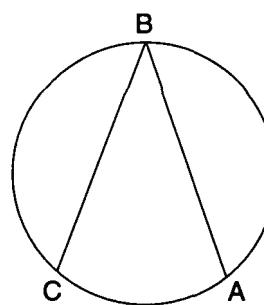


Find:

- a  $m\widehat{AB}$   
b  $m\angle ACB$   
c  $m\angle P$  to the nearest degree

**a 80; b 40; c 20**

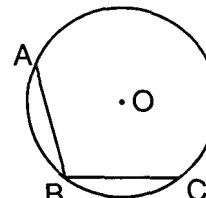
3738. The new corporate logo created by the design engineers at Magic Motors is shown in the accompanying diagram.



If chords  $\overline{BA}$  and  $\overline{BC}$  are congruent and  $m\widehat{BC} = 140$ , what is  $m\angle B$ ?

- (A) 40**                              (C) 140  
(B) 80                                (D) 280

3674. In the accompanying diagram of circle  $O$ ,  $m\widehat{ABC} = 150$ .

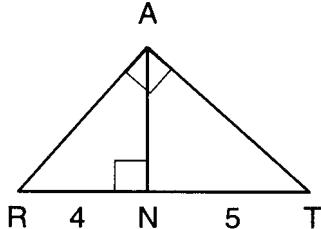


- What is  $m\angle ABC$ ?  
(A) 210                              (C) 95  
**(B) 105**                            (D) 75

### III. THE COORDINATE PLANE

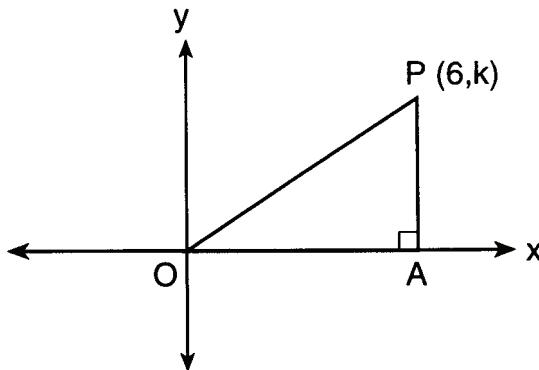
#### B. Areas in the Coordinate Plane

3549. In the accompanying diagram of right triangle  $RAT$ , altitude  $\overline{AN}$  divides hypotenuse  $\overline{RT}$  into segments with lengths of 4 and 5. Find the length of leg  $\overline{RA}$ .



**6**

1926. In the accompanying diagram of right triangle  $OAP$ , the coordinates of  $P$  are  $(6, k)$ . Find the value of  $k$  if the area of  $\triangle OAP$  is 12.



**4**

1911. The vertices of  $\triangle ABC$  are  $A(0,0)$ ,  $B(0,k)$ , and  $C(k,0)$ . The area of this triangle can be expressed as

$$\frac{k^2}{2} \quad (C)$$

$$\frac{k^2}{4} \quad (D)$$

1887. The vertices of  $\triangle ABC$  are  $A(9,0)$ ,  $B(-3,0)$ , and  $C(0,5)$ . What is the area of  $\triangle ABC$  in square units?

$$(A) 22.5 \quad (C) 45 \\ (B) 30 \quad (D) 60$$

1872. Find the area of a triangle whose vertices are  $(1,2)$ ,  $(8,2)$ , and  $(1,6)$ .

**14**

1867. The vertices of trapezoid  $ABCD$  are  $A(-3,0)$ ,  $B(-3,4)$ ,  $C(2,4)$ , and  $D(4,0)$ . What is the area of trapezoid  $ABCD$ ?

$$(A) 6 \quad (C) 28 \\ (B) 24 \quad (D) 48$$

1839. Find the area of the parallelogram whose vertices are  $(2,1)$ ,  $(7,1)$ ,  $(9,5)$ , and  $(4,5)$ .

**20**

1830. Find the number of square units in the area of the triangle whose vertices are points  $A(2,0)$ ,  $B(6,0)$ , and  $C(8,5)$ .

**10**

1828. The coordinates of the vertices of  $\triangle ABC$  are  $A(0,0)$ ,  $B(3,0)$ , and  $C(0,4)$ . What is the length of  $\overline{BC}$ ?

**5**

1797. Find the number of square units in the area of a triangle whose vertices are  $A(2,0)$ ,  $B(6,0)$ , and  $C(4,5)$ .

**10**

1747. The vertices of  $\triangle ABC$  are  $A(0,6)$ ,  $B(3,0)$ , and  $C(11,0)$ . What is the area of  $\triangle ABC$  in square units?

$$(A) 9 \quad (C) 24 \\ (B) 12 \quad (D) 33$$

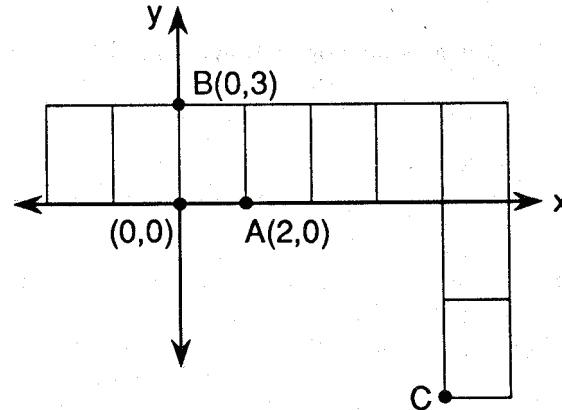
1737. If the coordinates of a parallelogram are  $Q(3,-2)$ ,  $R(7,-2)$ ,  $S(9,3)$ , and  $T(5,3)$ , the area of the parallelogram is

$$(A) 10 \quad (C) 30 \\ (B) 20 \quad (D) 40$$

1711. Find the number of square units in the area of the figure formed by the intersection of the lines  $y = 4$ ,  $x = 1$ ,  $y = 6$ , and  $x = 2$ .

**2**

1701. The accompanying diagram contains congruent rectangles that measure 2 units by 3 units. If the coordinates of  $A$  are  $(2,0)$  and the coordinates of  $B$  are  $(0,3)$ , find the coordinates of  $C$ .



**(8,-6)**

1678. Find the area of the triangle whose vertices are  $(0,8)$ ,  $(0,0)$ , and  $(7,0)$ .

**28**

1632. Find, in terms of  $\pi$ , the area of the circle whose equation is  $x^2 + y^2 = 9$ .

**9π**

1630. What is the area of the triangle whose vertices are  $(2,3)$ ,  $(6,3)$ , and  $(3,5)$ ?

**4**

1594. What is the area of the triangle whose vertices are  $(0,0)$ ,  $(3,0)$ , and  $(0,4)$ ?

**6**

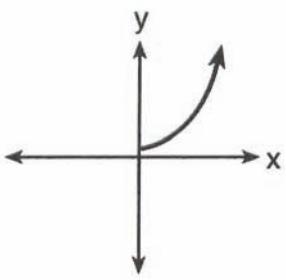
### 1. Analytic Geometry

#### i. Coordinate Area & Perimeter (Short Ans)

### III. THE COORDINATE PLANE

#### C. Transformations

4421. The accompanying graph shows the relationship between kinetic energy,  $y$ , and velocity,  $x$ .



The reflection of this graph in the line  $y = x$  is

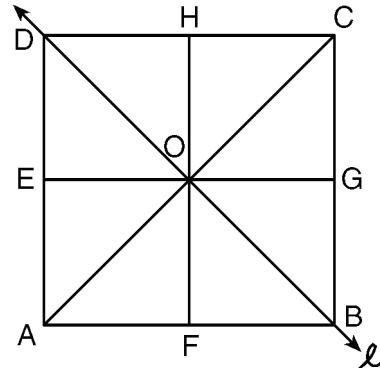
- (A)
- (B)
- (C)
- (D)

3244. A property *not* preserved under a line reflection is  
 (A) angle measure                      (C) distance  
 (B) collinearity                        (D) orientation

4325. Ms. Brewer's art class is drawing reflected images. She wants her students to draw images reflected in a line. Which diagram represents a correctly drawn image?

- (A)
- (B)
- (C)
- (D)

4125. In the accompanying diagram of square  $ABCD$ ,  $F$  is the midpoint of  $\overline{AB}$ ,  $G$  is the midpoint of  $\overline{BH}$ ,  $H$  is the midpoint of  $\overline{CD}$ , and  $E$  is the midpoint of  $\overline{DA}$ .



Find the image of  $\triangle EOA$  after it is reflected in line  $\ell$ . Is this isometry direct or opposite? Explain your answer.

**$\Delta HOC$  and opposite**

3268. What are the coordinates of the image of point  $A(-2, -5)$  after a reflection over the line  $y = -1$ ?  
 (-2, +3)

1924. What are the coordinates of  $P'$ , the image of  $P(2, -5)$  after a reflection in the  $y$ -axis?  
 (-2, -5)

### III. THE COORDINATE PLANE

#### C. Transformations

#### 1. Analytic Geometry

#### iii. Translations

3318. Main Street and Park Avenue intersect at an angle of  $74^\circ$ . Mr. Jones lives on Main Street, 50 meters from the intersection, and Mr. Smith lives on Park Avenue, 40 meters from the intersection. The triangle formed by the intersection and the houses is an acute triangle. Find, to the nearest meter, the distance between Mr. Jones' house and Mr. Smith's house.

**55**

3301. Triangle  $ABC$  has coordinates  $A(1,1)$ ,  $B(5,1)$ , and  $C(4,3)$ .

Given the transformations  $T$ ,  $U$ , and  $W$  described below:

$$\begin{aligned}T: (x,y) &\rightarrow (x,-y) \\U: (x,y) &\rightarrow (x-6,y+6) \\W: (x,y) &\rightarrow (-2x,-2y)\end{aligned}$$

a Graph  $\Delta ABC$  and graph and state the coordinates of its image  $\Delta A'B'C'$ , after transformation  $T$ .

b Graph and state the coordinates of  $\Delta A''B''C''$ , the image of  $\Delta ABC$  after transformation  $U$ .

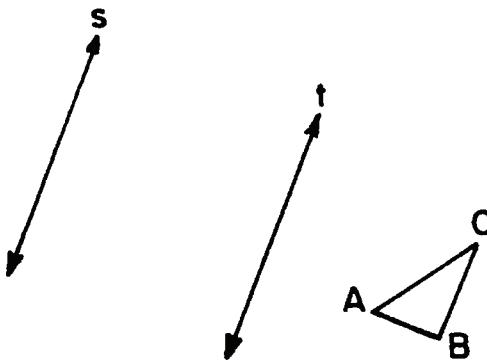
c Graph and state the coordinates of  $\Delta A'''B'''C'''$ , the image of  $\Delta ABC$  after transformation  $W$ .

d Which transformation,  $T$ ,  $U$ , or  $W$ , is not an isometry?

e Which transformation,  $T$ ,  $U$ , or  $W$ , does not preserve orientation?

- a  $A'(1,-1), B'(5,-1), C'(4,-3)$
- b  $A''(-5,7), B''(-1,7), C''(-2,9)$
- c  $A'''(-2,-2), B'''(-10,-2), C'''(-8,-6)$
- d  $W$
- e  $T$

3299. In the accompanying diagram, line  $s$  is parallel to line  $t$ . Which is equivalent to the composition of line reflections  $r_s \circ r_t(\Delta ABC)$ ?



- (A) a rotation
- (B) a line reflection
- (C) a translation
- (D) a glide-reflection

3292. The mapping which moves every point in the plane under the rule  $(x,y) \rightarrow (x+2,y-3)$  is called a

- (A) line reflection
- (B) rotation
- (C) translation
- (D) dilation

3280. What is the image of the point  $(5,-2)$  under the translation  $T_{2,1}$ ?

**(7,-1)**

3275. A transformation maps  $P(x,y)$  onto  $P'(x-2,y+3)$ . Under the same transformation find the coordinates of  $Q'$ , the image of  $Q(-4,1)$ .

**(-6,4)**

3266. If  $\ell$  and  $m$  are parallel lines, then  $r_\ell \circ r_m(\overline{AB})$  is equivalent to a

- (A) rotation
- (B) dilation
- (C) **translation**
- (D) glide-reflection

3258. If  $T$  is the transformation  $(x,y) \rightarrow (-x,2y)$ , find the image of  $P(-3,4)$  after the transformation  $T$ .

**(3,8)**

3255. Given  $A(8,5)$  and  $B(6,1)$  and the transformations  $T$ ,  $R$ , and  $S$  as described below:

$$\begin{aligned}T: (x,y) &\rightarrow (x+1,y-5) \\R: (x,y) &\rightarrow (y,x) \\S: (x,y) &\rightarrow (-x,y)\end{aligned}$$

a Graph  $\overline{AB}$  and its image  $\overline{A'B'}$  after the transformation  $T$ .

b Graph  $\overline{A''B''}$ , the image of  $\overline{AB}$  after the transformation  $R$ .

c Graph  $\overline{A'''B'''}$ , the image of  $\overline{AB}$  after the transformation  $S$ .

d Compare the slopes of the pairs of segments listed below and indicate whether these slopes are *equal*, *reciprocals*, *additive inverses*, or *negative reciprocals*.

- (1)  $\overline{AB}$  and  $\overline{A'B'}$
- (2)  $\overline{AB}$  and  $\overline{A''B''}$
- (3)  $\overline{AB}$  and  $\overline{A'''B'''}$

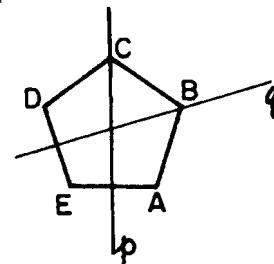
- d (1) equal

- (2) reciprocals

- (3) additive inverses

3246. In the accompanying figure,  $p$  and  $q$  are symmetry lines for a regular pentagon  $ABCDE$ .

Find  $r_q \circ r_p(B)$ .



**E**

3240. A translation maps  $P(4,-3)$  onto  $P'(0,0)$ . Find the coordinates of  $Q'$ , the image of  $Q(-2,1)$ , under the same translation.

**(-6,4)**

**III. THE COORDINATE PLANE**  
**F. Conic Sections and Exponential Functions**

**1. Analytic Geometry**  
**i. Equations of Parabolas**

1869. a On graph paper, draw the graph of the equation  $y = -x^2 + 6x - 8$  for all values of  $x$  in the interval  $0 \leq x \leq 6$ .

b What is the maximum value of  $y$  in the equation  $y = -x^2 + 6x - 8$ ?

c Write an equation of the line that passes through the turning point and is parallel to the  $x$ -axis.

**b** 1; **c**  $y = 1$

1855. What is the turning point of the graph of the function  $y = x^2 - 6x + 2$ ?

- (A) (3, -7) (C) (3, 11)  
 (B) (-3, -7) (D) (-3, 11)

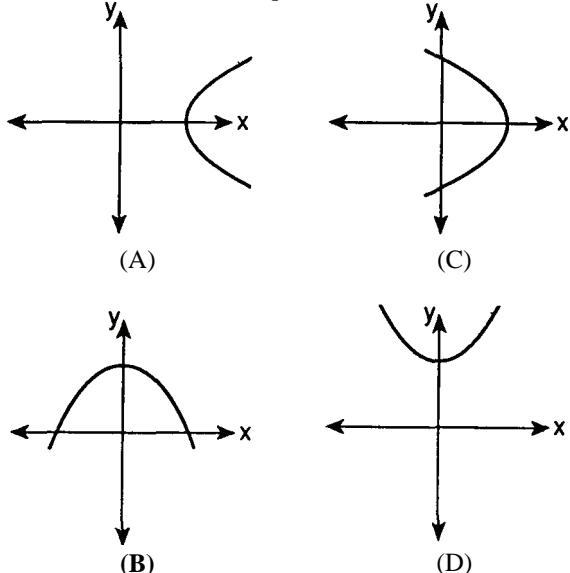
1845. Which equation represents the axis of symmetry of the graph of the equation  $y = 2x^2 + 7x - 5$ ?

- (A)  $x = -\frac{5}{4}$   
 (B)  $x = \frac{5}{4}$   
 (C)  $x = \frac{7}{4}$   
**(D)**  $x = -\frac{7}{4}$

1833. Which is an equation of the axis of symmetry of the graph of the equation  $y = x^2 - 6x + 2$ ?

- (A)  $x = -3$  (C)  $x = 3$   
 (B)  $y = -3$  (D)  $y = 3$

1824. Which graph represents the equation  $y = -x^2 + 4$ ?



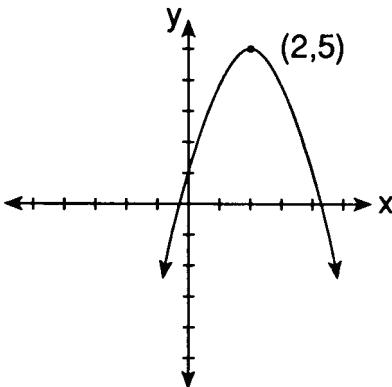
1814. Which equation represents the axis of symmetry of the graph of the equation  $y = x^2 - 6x + 5$ ?

- (A)  $x = -3$  (C)  $x = 3$   
 (B)  $y = -3$  (D)  $y = 3$

1792. What are the coordinates of the turning point of the parabola whose equation is  $y = x^2 - 2x - 3$ ?

- (A) (1, -4) (C) (-1, 0)  
 (B) (1, 2) (D) (-1, -2)

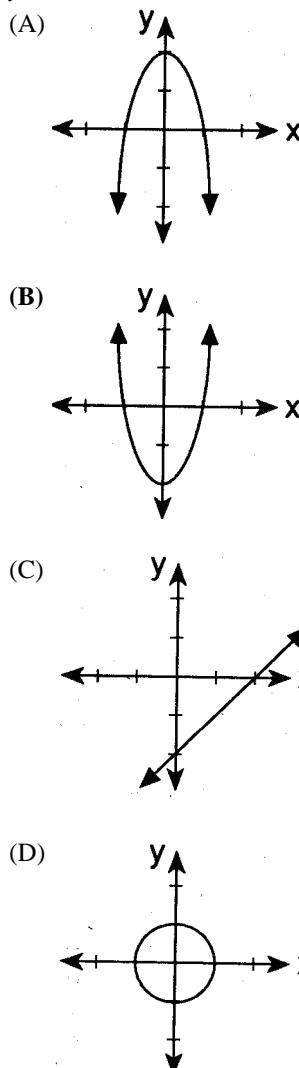
1811. The parabola shown in the diagram is reflected in the  $x$ -axis.



What is the image of the turning point after the reflection?

- (A) (2, -5) (C) (-2, -5)  
 (B) (-2, 5) (D) (5, 2)

1803. Which graph could represent the equation  $y = x^2 - 4$ ?



### III. THE COORDINATE PLANE

#### F. Conic Sections and Exponential Functions

5325. Which equation represents an ellipse?

- (A)  $3x^2 = 4 - 5y^2$       (C)  $6x^2 = 9 + 8y$   
 (B)  $4x^2 = 9 - 4y$       (D)  $xy = 12$

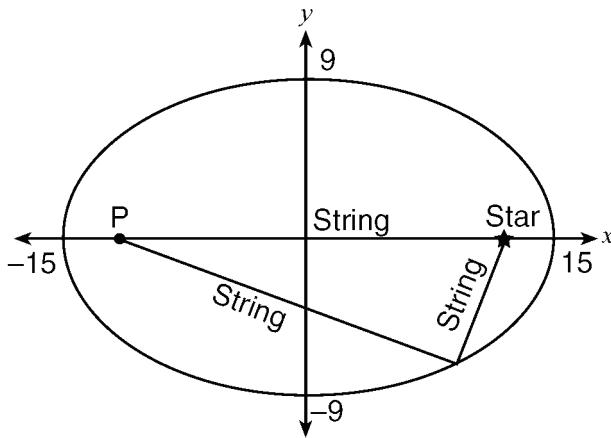
4316. An object orbiting a planet travels in a path represented by the equation  $3(y + 1)^2 + 5(x + 4)^2 = 15$ . In which type of pattern does the object travel?

- (A) hyperbola      (C) circle  
 (B) ellipse      (D) parabola

4297. Which equation, when graphed on a Cartesian coordinate plane, would best represent an elliptical racetrack?

- (A)  $3x^2 + 10y^2 = 288,000$       (C)  $3x + 10y = 288,000$   
 (B)  $3x^2 - 10y^2 = 288,000$       (D)  $30xy = 288,000$

4283. The accompanying diagram shows the construction of a model of an elliptical orbit of a planet traveling around a star. Point P and the center of the star represent the foci of the orbit.



Which equation could represent the relation shown?

- $\frac{x^2}{81} + \frac{y^2}{225} = 1$        $\frac{x^2}{15} + \frac{y^2}{9} = 1$   
 (A)      (C)  
 $\frac{x^2}{225} + \frac{y^2}{81} = 1$        $\frac{x^2}{15} - \frac{y^2}{9} = 1$   
 (B)      (D)

4199. A new planet is found that orbits the sun in a circular path. If  $a$ ,  $b$ , and  $c$  are all positive and unequal, which of the following could represent the motion of the planet in the  $xy$ -plane if the sun is at the origin?

- (A)  $ax^2 + by^2 = c^2$   
 (B)  $ax^2 - by^2 = c^2$   
 (C)  $x^2 + y^2 = c^2$   
 (D)  $by = ax + b$

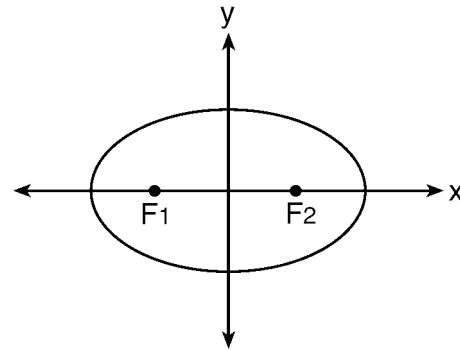
4176. A planet moves through a large coordinate grid in the solar system according to the equation  $3x^2 + 2y^2 = 12$ . What is the shape of the orbit of this planet?

- (A) circle      (C) parabola  
 (B) hyperbola      (D) ellipse

### 1. Analytic Geometry

#### iv. Ellipses in the Form $ax^2 + bx^2 = c$

4104. The accompanying diagram shows the elliptical orbit of a planet. The foci of the elliptical orbit are  $F_1$  and  $F_2$ .



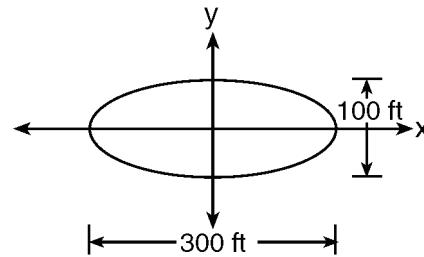
If  $a$ ,  $b$ , and  $c$  are all positive and  $a \neq b \neq c$ , which equation could represent the path of the planet?

- (A)  $ax^2 - by^2 = c^2$       (C)  $y = ax^2 + c^2$   
 (B)  $ax^2 + by^2 = c^2$       (D)  $x^2 + y^2 = c^2$

4092. A commercial artist plans to include an ellipse in a design and wants the length of the horizontal axis to equal 10 and the length of the vertical axis to equal 6. Which equation could represent this ellipse?

- (A)  $9x^2 + 25y^2 = 225$       (C)  $x^2 + y^2 = 100$   
 (B)  $9x^2 - 25y^2 = 225$       (D)  $3y = 20x^2$

4075. The accompanying diagram represents the elliptical path of a ride at an amusement park.



Which equation represents this path?

- $x^2 + y^2 = 300$        $\frac{x^2}{150^2} + \frac{y^2}{50^2} = 1$   
 (A)      (C)

- $y = x^2 + 100x + 300$        $\frac{x^2}{150^2} - \frac{y^2}{50^2} = 1$   
 (B)      (D)

4032. Which equation, when graphed, is an ellipse?

- (A)  $2x^2 + 4 = 2y^2$       (C)  $4x^2 + 4y^2 = 25$   
 (B)  $xy = 6$       (D)  $7x^2 + 3y^2 = 9$

3652. The graph of which equation forms an ellipse?

- (A)  $x^2 - y^2 = 9$       (C)  $2x^2 + y^2 = 8$   
 (B)  $2x^2 + 2y^2 = 8$       (D)  $xy = -8$

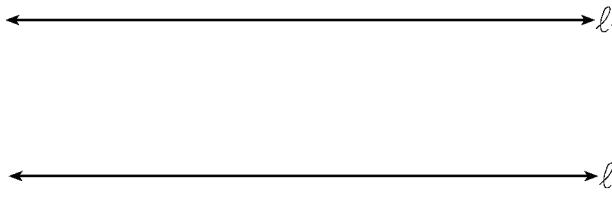
3500. When drawn on a set of axes, which equation is an ellipse?

- (A)  $2x^2 + y = 12$       (C)  $2x^2 + 2y^2 = 12$   
 (B)  $2x^2 + y^2 = 12$       (D)  $2x^2 - y^2 = 12$

### III. THE COORDINATE PLANE

#### G. Locus of Points

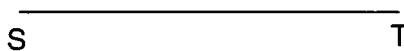
4052. In the accompanying diagram, line  $\ell_1$  is parallel to line  $\ell_2$ .



Which term describes the locus of all points that are equidistant from line  $\ell_1$  and line  $\ell_2$ ?

- (A) line                              (C) point  
(B) circle                              (D) rectangle

3560. Construct an equilateral triangle using line segment  $\overline{ST}$  as one side.



#### Construction

3554. The number of points equidistant from two parallel lines and also equidistant from two points on one of the given lines is *exactly*

- (A) 1                                    (C) 3  
(B) 2                                    (D) 4

1950. Which equation represents a circle whose center is  $(4, -5)$  and whose radius is 8?

- (A)  $(x + 4)^2 + (y - 5)^2 = 64$       (C)  $(x + 4)^2 + (y - 5)^2 = 8$   
(B)  $(x - 4)^2 + (y + 5)^2 = 64$       (D)  $(x - 4)^2 + (y + 5)^2 = 8$

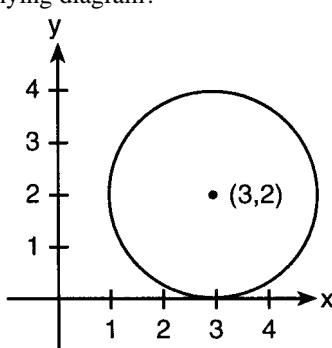
1946. Which equation represents the locus of points equidistant from points  $(4, 2)$  and  $(8, 2)$ ?

- (A)  $x = 6$                             (C)  $x = 12$   
(B)  $y = 6$                                    (D)  $y = 12$

1937. Which equation represents the locus of points equidistant from the lines whose equations are

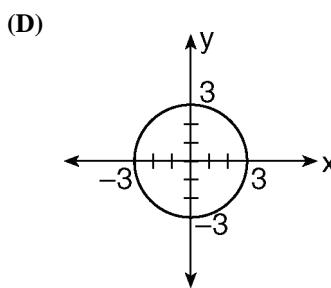
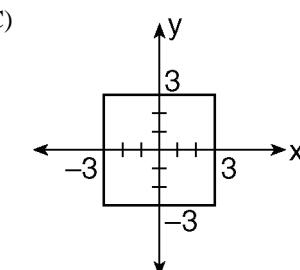
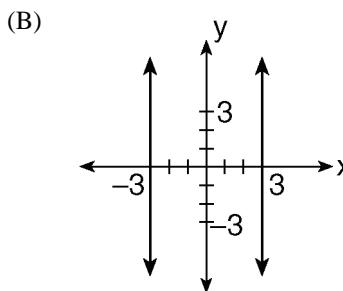
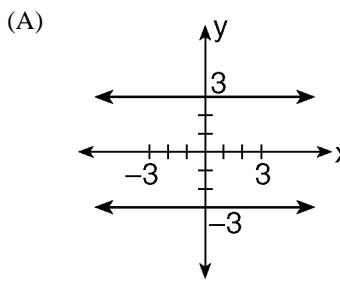
- $y = 3x + 8$  and  $y = 3x - 6$ ?
- (A)  $y = 3x - 1$                             (C)  $y = 3x - 4$   
(B)  $y = 3x + 1$                               (D)  $y = 3x + 4$

1929. Which equation is represented by the graph drawn in the accompanying diagram?



- (A)  $(x + 3)^2 + (y + 2)^2 = 4$       (C)  $(x + 3)^2 + (y + 2)^2 = 2$   
(B)  $(x - 3)^2 + (y - 2)^2 = 2$       (D)  $(x - 3)^2 + (y - 2)^2 = 4$

3760. Which graph represents the locus of points 3 units from the origin?



1916. Which is an equation of the locus of points equidistant from points  $(-2, 0)$  and  $(4, 0)$ ?

- (A)  $x = 1$                                     (C)  $y = 1$   
(B)  $x = -1$                                     (D)  $y = -1$

1909. The equation of the locus of points 5 units from the origin is

- (A)  $x^2 + y^2 = 5$                               (C)  $x = 5$   
(B)  $x^2 + y^2 = 25$                               (D)  $y = 5$

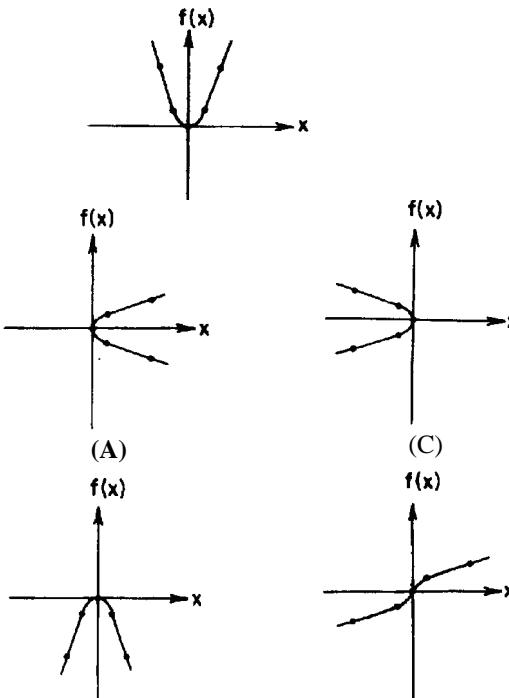
1895. What is the locus of points at a given distance from a line?

- (A) 1 point                                      (C) 1 circle  
(B) 2 points                                      (D) 2 parallel lines

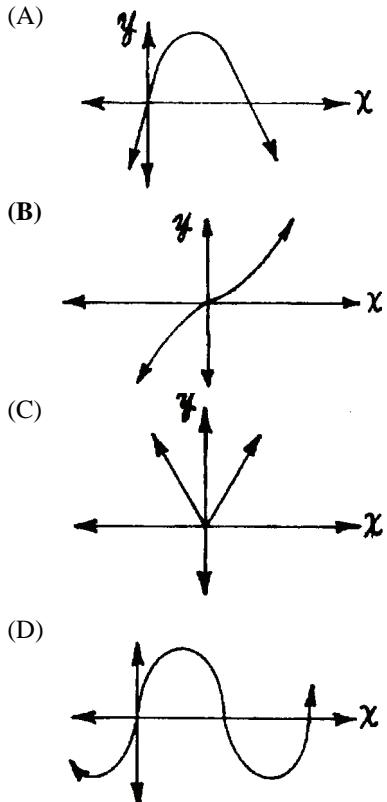
1885. Which equation represents the locus of points equidistant from points  $(1, 1)$  and  $(7, 1)$ ?

- (A)  $x = 4$                                       (C)  $x = -4$   
(B)  $y = 4$     (D)  $y = -4$

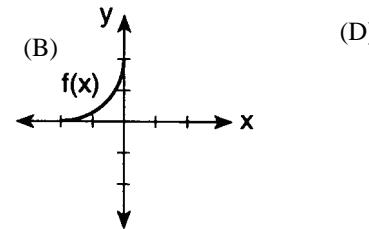
2559. In the diagram below, the function  $f(x) = x^2$  is represented graphically. Which graph represents the inverse of  $f(x)$ ?



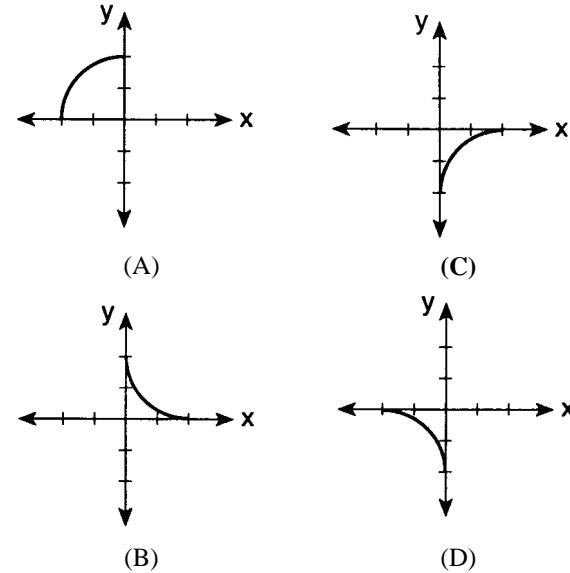
2452. Which function represented in the graph below has an inverse which is also a function?



3065. The accompanying diagram represents the graph of  $f(x)$ .



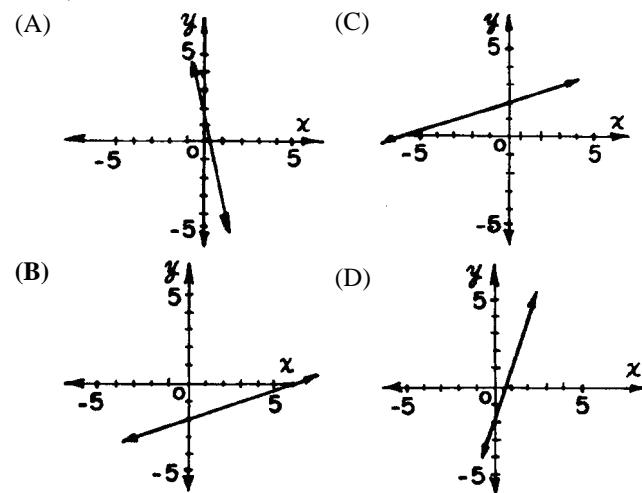
Which graph below represents  $f^{-1}(x)$ ?



2311. What is the inverse of the function  $y - 2 = 3x$ ?

- (A)  $y = \frac{2-x}{3}$   
 (B)  $y = \frac{2x}{3}$   
 (C)  $y = 3x - 2$   
 (D)  $y = \frac{x-2}{3}$

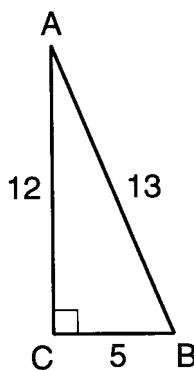
2382. Which is the graph of the inverse of the function  $y = 3x + 6$ ?



## V. TRIGONOMETRY (Optional)

### A. Trigonometry of the Right Triangle

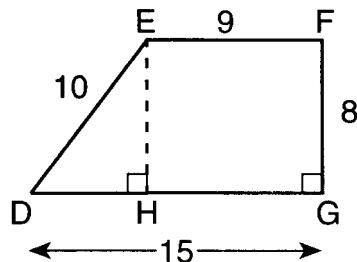
2013. In the accompanying diagram of right triangle  $ABC$ , legs  $AC$  and  $BC$  are 12 and 5, respectively, and hypotenuse  $AB$  is 13.



What is  $\tan B$ ?

- (A)  $\frac{12}{5}$   
 (B)  $\frac{12}{13}$   
 (C)  $\frac{5}{13}$   
 (D)  $\frac{5}{12}$

2009. In the accompanying diagram, altitude  $\overline{EH}$  is drawn in trapezoid  $DEFG$ ,  $DE = 10$ ,  $EF = 9$ ,  $FG = 8$ , and  $GD = 15$ .



What is  $m\angle D$  to the nearest degree?

- (A) 37  
 (C) 60  
 (B) 53  
 (D) 80

1992. If  $\tan A = 0.4548$ , find the measure of  $\angle A$  to the nearest degree.

24

1988. The hypotenuse of right triangle  $ABC$  is 10 and  $m\angle A = 60$ . What is the measure, to the nearest tenth, of the leg opposite  $\angle A$ ?

- (A) 5.0  
 (C) 7.1  
 (B) 5.8  
 (D) 8.7

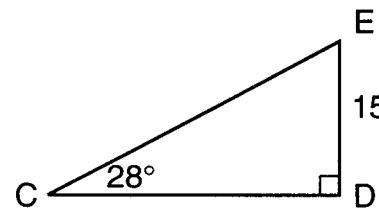
1986. If  $\cos x = 0.8$ , what is the value of  $\sin x$ ?

- (A) 1.0  
 (C) 0.6  
 (B) 0.2  
 (D) 0.4

1981. If  $\sin A = 0.3642$ , find the measure of  $\angle A$  to the nearest degree.

21

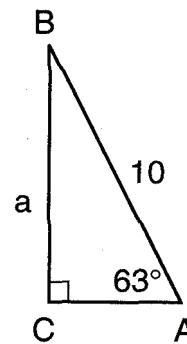
2008. In the accompanying diagram of  $\triangle CDE$ ,  $m\angle D = 90$ ,  $m\angle C = 28$ , and  $ED = 15$ .



Which equation can be used to find  $CD$ ?

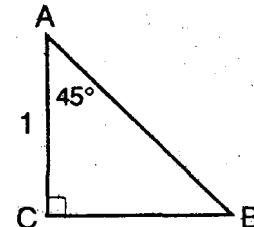
- (A)  $\sin 28^\circ = \frac{15}{CD}$   
 (B)  $\sin 28^\circ = \frac{CD}{15}$   
 (C)  $\tan 28^\circ = \frac{15}{CD}$   
 (D)  $\tan 28^\circ = \frac{CD}{15}$

2004. In right triangle  $ABC$ ,  $m\angle C = 90$ ,  $m\angle A = 63$  and  $AB = 10$ . If  $BC$  is represented by  $a$ , then which equation can be used to find  $a$ ?



- (A)  $\sin 63^\circ = \frac{a}{10}$   
 (B)  $a = 10 \cos 63^\circ$   
 (C)  $\tan 63^\circ = \frac{a}{10}$   
 (D)  $a = \tan 27^\circ$

1982. In the accompanying diagram of right triangle  $ABC$ ,  $m\angle C = 90$ ,  $m\angle A = 45$ , and  $AC = 1$ . Find, in radical form, the length of  $AB$ .



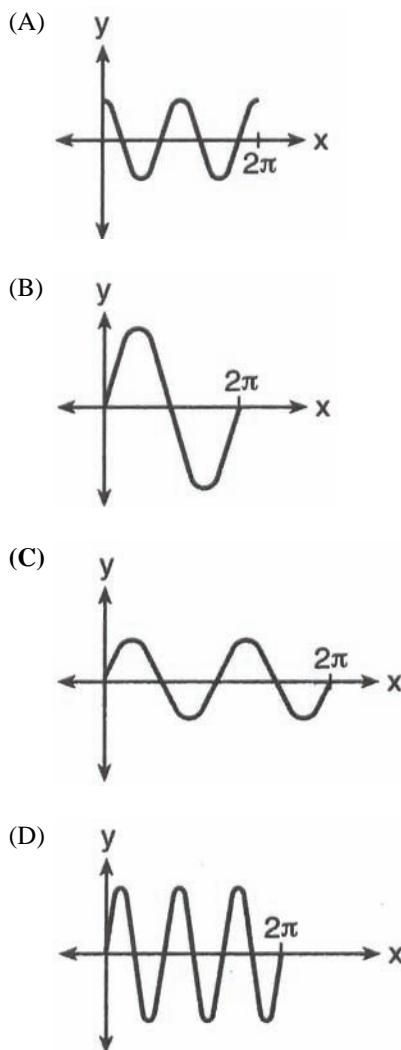
$\sqrt{2}$

## V. TRIGONOMETRY (Optional) C. Graphing Trigonometric Functions

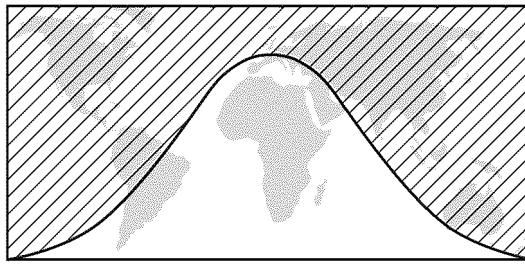
## 1. Trigonometry

### ii. The Graphs of Sine, Cosine & Tangent

4419. Which graph represents a sound wave that follows a curve whose period is  $\pi$  and that is in the form  $y = a \sin bx$ ?



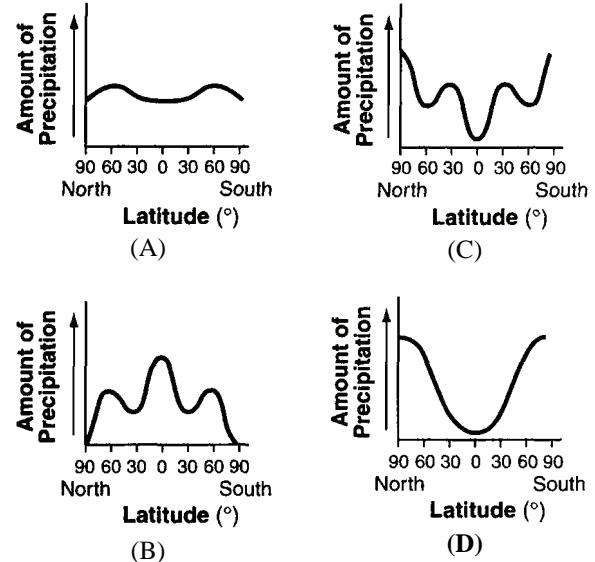
4275. The shaded portion of the accompanying map indicates areas of night, and the unshaded portion indicates areas of daylight at a particular moment in time.



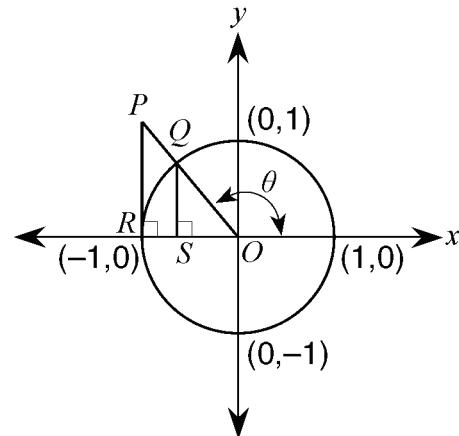
Which type of function best represents the curve that divides the area of night from the area of daylight?

- (A) quadratic (C) tangent  
(B) cosine (D) logarithmic

4310. The graphs below show the average annual precipitation received at different latitudes on Earth. Which graph is a translated cosine curve?



4299. In the accompanying diagram,  $\overline{PR}$  is tangent to circle  $O$  at  $R$ ,  $\overline{QS} \perp \overline{OR}$ , and  $\overline{PR} \perp \overline{OR}$ .



Which measure represents  $\sin \theta$ ?

- (A)  $SO$  (C)  $PR$   
(B)  $RO$  (D)  $QS$

4013. a On the same set of axes, sketch and label the graphs of the equations  $y = \frac{1}{2}\sin 2x$  and  $y = -2 \cos x$  in the interval  $0 \leq x \leq 2\pi$ .

b Using the graphs drawn in part a, find all values of  $x$  in the interval  $0 \leq x \leq 2\pi$  that satisfy the equation  $\frac{1}{2}\sin 2x = -2 \cos x$ .

$$b \quad \frac{\pi}{2}, \frac{3\pi}{2}$$

3001. Between  $-2\pi$  and  $2\pi$ , the graph of the equation  $y = \cos x$  is symmetric with respect to

- (A) the  $y$ -axis (C) the origin  
(B) the  $x$ -axis (D)  $y = x$