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I. FUNCTIONS, GRAPHS AND LIMITS

A. Rational Functions

1. Functions

1071. What is the domain of the function \( f(x) = \frac{x^2 - 16}{x^2 + 7x + 12} ? \)

(A) \((-4, 4)\)  (D) \((-\infty, \infty)\)
(B) \(x \neq -3, -4\)  (E) \(x \neq -4, 4\)
(C) \(x \neq -3, -4, 4\)

1097. The domain of the function \( f(x) = \frac{1}{\sqrt{4 - x}} \) is

(A) \(x \geq 0\)  (D) \(x \geq 4\)
(B) \(x > 4\)  (E) \(x \leq 4\)
(C) \(x < 4\)

The graph of \(y^2 - 5y - 1 = x^2\) is a(n)

(A) circle  (D) line
(B) ellipse  (E) parabola
(C) hyperbola

1168. If the zeros of \(f(x)\) are \(x = -2\) and \(x = 3\), then \(f\left(\frac{x}{2}\right)\) are \(x = \)

(A) \(-1, \frac{3}{2}\)  (D) \(-3, \frac{9}{2}\)
(B) \(-2, 3\)  (E) \(-4, 6\)
(C) \(-2, 6\)

If \(f(x) = 3 - x\) and \(g(x) = \sqrt{x - 5}\), then \(f(g(-2)) = \)

1244. \( \) (A) 0  (D) \(\sqrt{2}\)
(B) \(4 - \sqrt{7}\)  (E) Undefined
(C) \(\sqrt{7}\)

1524. The function \(f(x) = \frac{x^2 + 9x - 90}{x^2 + 19x + 60}\) has a removable discontinuity at

(A) \(-15\)  (D) 7
(B) \(-4\)  (E) 15
(C) 6

1582. If \(f(x) = \frac{3}{x^2 - 2}\) and \(g(x) = 4x\), then \(g(f(3)) = \)

(A) \(\frac{3}{7}\)  (D) \(\frac{12}{7}\)
(B) \(\frac{2}{3}\)  (E) \(\frac{14}{7}\)
(C) \(\frac{12}{3}\)

Which of the following functions is represented by the graph above?

(A) \(f(x) = \left|\frac{3}{2}x + 1\right|\)  (D) \(f(x) = \frac{3}{x} + 1\)
(B) \(f(x) = \left|\frac{3}{2}x + 1\right|\)  (E) \(f(x) = \frac{3}{2}x - 1\)
(C) \(f(x) = -\frac{3}{2}x + 1\)

1525. The function \(h(x) = \frac{f(x)}{g(x)}\) is discontinuous whenever

(A) \(g(x) = 1\)  (D) \(g(x) = 0\)
(B) \(g(x)\) is negative  (E) \(f(x) = g(x)\)
(C) \(f(x) = 0\)

A polynomial has a relative maximum at \((-5, 4)\), a relative minimum at \((1, -7)\) and a relative maximum at \((4, 2)\), and no other critical points. How many real roots does the polynomial have?

(A) 2  (D) 5
(B) 3  (E) 6
(C) 4
II. DERIVATIVES
1. Definitions

If the graph of \( f(x) \) is symmetric with respect to the line \( y = x \) and is differentiable everywhere, then the graph of \( f'(x) \) is:

(A) Symmetric with respect to the line \( y = 0 \)
(B) Symmetric with respect to the line \( x = 0 \)
(C) A parabola
(D) A hyperbola
(E) None of the above

If \( f(x) \) is a continuous differentiable function and \( f'(a) = 0 \), what can always be said about the graph of \( f(x) \) at \( a \)?

I. The line tangent to the curve at \( a \) is horizontal.
II. The curve is at a local maximum or minimum
III. There is an inflection point at \( a \).

(A) I only
(B) I and II
(C) II and III

Base your answers to questions 1241 through 1238 on the graph below of \( f(x) \).

Where is \( f'(x) \) undefined?

1241. (A) \( x = 1 \)
(B) \( x = 2 \)
(C) \( x = 0 \)

(D) All of the above
(E) None of the above

Where is \( f''(x) > 0? \)

1239. (A) \( (\infty, 0) \) and \( (1, 2) \)
(B) \( (0, 1) \) and \( (2, \infty) \)
(C) \( (0, 3) \)

(D) \( (\infty, -\frac{1}{2}) \) and \( (3, 0) \)
(E) \( (\infty, -\frac{1}{2}) \) and \( (\frac{3}{2}, \infty) \)

Base your answers to questions 607 through 605 on the graph below of \( f(x) \).

607. At which point is \( f'(x) = 0 \) and \( f''(x) > 0 \)?

(A) \( A \)
(B) \( B \)
(C) \( C \)

(D) \( D \)
(E) \( E \)

606. At which point is \( f'(x) < 0 \) and \( f''(x) = 0 \)?

(A) \( A \)
(B) \( B \)
(C) \( C \)

(D) \( D \)
(E) \( E \)
There are two lines through the point (1, –2) that are tangential to the parabola \( y = x^2 + 2 \). Find the \( x \)-coordinates of these points of intersection.

(A) \( x = –0.873, 6.873 \)  
(B) \( x = –1.236, 3.236 \)  
(C) \( x = –2.327, 3.327 \)

If \( 2x – 5y = 23 \) is the equation of the line normal to the graph of \( f \) at the point (14, 1), then \( f'(14) = \)

(A) \( \frac{2}{5} \)  
(B) \( \frac{5}{2} \)  
(C) \( -\frac{2}{5} \)

The line tangent to the graph of \( y = –x^2 \) at the point (1, –1) intersects both \( x \) and \( y \) axes. What is the area of the triangle formed by this tangent line and coordinate axes?

(A) 1  
(B) 2  
(C) 2.25

What is the slope of the line tangent to the curve \( y = e^x \) at \( x = e \)?

(A) \( e \)  
(B) 1  
(C) \( e^\pi \)

Find the equation to the line tangent to the polar curve \( r = 2\cos (\frac{\pi}{2} \theta) \) at \((0, \pi)\).

(A) \( y = –x + \frac{\pi}{2} \)  
(B) \( y = –\frac{\pi}{2} \)  
(C) \( y = \frac{\pi}{2} \)

Find the equation of the line normal to the curve \( x^2 + y^2 = 1 \) at \((\cos \frac{5\pi}{4}, \sin \frac{5\pi}{4})\).

(A) \( y = –x + \frac{\pi}{2} \)  
(B) \( y = –\frac{\pi}{2} \)  
(C) \( y = \frac{\pi}{2} \)

Which of the following is the equation of the line tangent to the curve \( f(x) = 2 \sin (x) \) at \( x = \pi \)?

(A) \( y = –2x + 2\pi \)  
(B) \( \frac{y}{2} = x – \pi \)  
(C) \( y = 2x – 2\pi \)

The tangent line to the curve \( y = x^3 – 2x + 4 \) at the point (0,4) has an \( x \)-intercept at

(A) \( 0, 0 \)  
(B) \( -2, 0 \)  
(C) \( 2, 0 \)

If \( f(x) = 3x^2 – 5 \), what is the equation for the tangent line to the curve when \( x = 1 \)?

(A) \( y + 2 = 6(x – 1) \)  
(B) \( y – 2 = –12(x – 1) \)  
(C) \( y – 2 = 6(x – 1) \)

982. If \( f(x) = 2x^3 – 4x^2 – 6x + 1 \), then the equation for the normal line to the curve perpendicular to \( y + 3 = 2(x – 1) \) is

(A) \( y + 2x + 15 = 0 \)  
(B) \( y + 2x – 15 = 0 \)  
(C) \( y – 2x + 15 = 0 \)
II. DERIVATIVES

4. Applications

1. Motion

Base your answers to questions 586 through 584 on the graph below, which shows the velocity of an object moving along a straight line during the time interval 0 ≤ t ≤ 7.

586. At what time(s) does the object change direction?
(A) t = 3 and t = 5  (D) t = 6
(B) t = 5 and t = 6  (E) t = 5
(C) t = 3

584. At what time(s) does the object reach its maximum acceleration?
(A) 1 < t < 5  (D) t = 6
(B) 0 < t < 2  (E) t = 5
(C) t = 3

519. The displacement from the origin of a particle moving on a line is given by s = t^4 - 2t^3. The maximum displacement during the time interval −1 ≤ t ≤ 3 is approximately
(A) 13.966  (D) 18
(B) 18.685  (E) 34.604
(C) 27.000

Base your answers to questions 504 through 500 on the information below.

A particle moves along a horizontal line and its position at time t is s = \( \frac{1}{2}t^4 - 2t^3 + 2t^2 + 5 \).

504. The position of the particle is always increasing for
(A) t > 0  (D) t < 0
(B) t > 1  (E) 0 < t < 2
(C) t > 2

502. The velocity is increasing when
(A) t < 0
(B) t > 2
(C) 0 < t < 1 or t > 2
(D) \( \frac{\sqrt{3} - 3}{3} < t < \frac{\sqrt{3} + 3}{3} \)
(E) \( t < -\frac{\sqrt{3} - 3}{3} \) and \( t > \frac{\sqrt{3} + 3}{3} \)

500. The particle is at rest when t is equal to
(A) 0  (D) 0 and 2
(B) 1 and 2  (E) 0, 1, and 2
(C) 0 and 1

Base your answers to questions 499 through 496 on the information below.

The position of a particle moving along a straight line is given by s = t^3 - 9t^2 + 24t - 6.

499. The speed of the particle is decreasing for
(A) t < 1  (D) t < 1 and t > 2
(B) t > 2  (E) all t
(C) t < 3

498. The acceleration is positive
(A) when t < 3  (D) for 2 < t < 4
(B) when t > 3  (E) for all t, t ≠ 3
(C) for 2 < t < 3

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775. The sides of an equilateral triangle are increasing at the rate of 12 in/sec. How fast is the triangle’s area increasing when the sides of the triangle are each 2 inches long?

\( \text{(A)} \ 6\sqrt{3} \text{ ft/sec} \quad \text{(D)} \ 36\sqrt{3} \text{ ft/sec} \\
\text{(B)} \ 12\sqrt{3} \text{ ft/sec} \quad \text{(E)} \ 48\sqrt{3} \text{ ft/sec} \\
\text{(C)} \ 24\sqrt{3} \text{ ft/sec} \\

740. A boat is being pulled toward a dock by a rope attached to its bow through a pulley on the dock 3 feet above the bow. If the rope is hauled in at a rate of 5 ft/sec, how fast is the boat approaching the dock when its 4 feet from the dock?

\( \text{(A)} \ 2 \text{ ft/sec} \quad \text{(D)} \ 5 \text{ ft/sec} \\
\text{(B)} \ 3 \text{ ft/sec} \quad \text{(E)} \ 6 \text{ ft/sec} \\
\text{(C)} \ 4 \text{ ft/sec} \\

716. An inverted conical container has a diameter of 36 in and a depth of 12 in. If water is flowing out of the vertex of the container at a rate of \( 15\pi \text{ in}^3/\text{sec} \), how fast is the depth of the water dropping when the height is 3 inches?

\( \text{(A)} \ \frac{15}{2} \text{ in/s} \quad \text{(D)} \ \frac{25}{4} \text{ in/s} \\
\text{(B)} \ \frac{15}{4} \text{ in/s} \quad \text{(E)} \ \frac{20}{7} \pi \text{ in/s} \\
\text{(C)} \ \frac{10}{9} \text{ in/s} \\

597. A foot tall rectangular box’s length is increasing at the rate of 0.25 feet/sec and its width is decreasing at 0.5 feet/sec. When the length is 0.8 feet and its width is 1 foot, the volume of the box is changing at

\( \text{(A)} \ -0.25 \text{ ft}^3/\text{sec} \quad \text{(D)} \ 0.25 \text{ ft}^3/\text{sec} \\
\text{(B)} \ -0.15 \text{ ft}^3/\text{sec} \quad \text{(E)} \ 0.4 \text{ ft}^3/\text{sec} \\
\text{(C)} \ 0.15 \text{ ft}^3/\text{sec} \\

555. The height \( h \) and radius \( r \) of a cylinder both increase at the rate of 1 in/sec. How fast does the surface area increase?

\( \text{(A)} \ 2\pi (r + h) \quad \text{(D)} \ 4\pi + 2\pi h \\
\text{(B)} \ 2\pi (3r + h) \quad \text{(E)} \ 4\pi r \\
\text{(C)} \ 6\pi \\

553. An inverted circular cone with its vertex down has a depth of 10 in and a radius at the top of 4 in. Water is leaking out so that the water level is falling at the rate of 1 in/hr. How fast the water leaking out of the cone when the water is 6 in deep?

\( \text{(A)} \ 1.92\pi \text{ in}^3/\text{hr} \quad \text{(D)} \ 3.8\pi \text{ in}^3/\text{hr} \\
\text{(B)} \ 2.4\pi \text{ in}^3/\text{hr} \quad \text{(E)} \ 5.76\pi \text{ in}^3/\text{hr} \\
\text{(C)} \ 3\pi \text{ in}^3/\text{hr} \\

551. A spherical balloon is being filled with helium at the rate of 9 ft\(^3\)/min. How fast is the surface area increasing when the volume is \( 36\pi \text{ ft}^3 \)?

\( \text{(A)} \ 2 \text{ ft}^2/\text{min} \quad \text{(D)} \ 3\pi \text{ ft}^2/\text{min} \\
\text{(B)} \ 2\pi \text{ ft}^2/\text{min} \quad \text{(E)} \ 6 \text{ ft}^2/\text{min} \\
\text{(C)} \ 3 \text{ ft}^2/\text{min} \\

470. A 5-foot-tall person is walking at a rate of 3 ft/sec away from a street lamp that is 16 feet tall. How fast is the length of her shadow changing?

\( \text{(A)} \ \frac{3}{11} \text{ ft/sec} \quad \text{(D)} \ 11 \text{ ft/sec} \\
\text{(B)} \ \frac{15}{11} \text{ ft/sec} \quad \text{(E)} \ 15 \text{ ft/sec} \\
\text{(C)} \ 3 \text{ ft/sec} \\

468. Milk spilled from a carton spreads in a circle whose circumference increases at a rate of 20 ft/sec. How fast is the area of the spill increasing when the circumference of the circle is \( 81\pi \text{ ft} \)?

\( \text{(A)} \ 810 \text{ ft}^2/\text{sec} \quad \text{(D)} \ 180 \text{ ft}^2/\text{sec} \\
\text{(B)} \ 360 \text{ ft}^2/\text{sec} \quad \text{(E)} \ 90 \text{ ft}^2/\text{sec} \\
\text{(C)} \ 270 \text{ ft}^2/\text{sec} \\

212. [Calculator] The best approximation to the increase in volume of a sphere when the radius is increased from 2 to 2.1 is

\( \text{(A)} \ 9 \quad \text{(D)} \ 6 \\
\text{(B)} \ 8 \quad \text{(E)} \ 5 \\
\text{(C)} \ 7 \)
II. DERIVATIVES
4. Applications

Base your answers to questions 538 through 537 on the graph below of \( f'(x) \).

538. \( f \) has a point of inflection at \( x = \)
\( \text{(A) } 0.5 \quad \text{(D) } 0.5 \text{ and } 3 \)
\( \text{(B) } 3 \quad \text{(E) } 1 \text{ and } 5 \)
\( \text{(C) } 5 \)

537. \( f' \) has a local minimum at \( x = \)
\( \text{(A) } 0 \quad \text{(D) } 3 \)
\( \text{(B) } 0.5 \quad \text{(E) } 5 \)
\( \text{(C) } 1 \)

480. On the closed interval \([0, 2\pi]\), the maximum value of the function \( f(x) = 5\sin x - 12\cos x \) is
\( \text{(A) } 0 \quad \text{(D) } 12 \)
\( \text{(B) } \frac{60}{13} \quad \text{(E) } 13 \)
\( \text{(C) } 5 \)

225. The minimum value of the slope of the curve
\( y = x^6 - x^4 + 3x \) is
\( \text{(A) } -3.025 \quad \text{(D) } 2.595 \)
\( \text{(B) } -2.595 \quad \text{(E) } 3.025 \)
\( \text{(C) } 0 \)

215. If \( f \) is differentiable and difference quotients underestimate the slope of \( f \) at \( x = a \) for all \( h > 0 \), which of the following must be true?
\( \text{(A) } f'' < 0 \quad \text{(D) } f'' > 0 \)
\( \text{(B) } f' < 0 \quad \text{(E) } \text{None of the above} \)
\( \text{(C) } f' > 0 \)

320. The function is concave downward for which interval?
\( \text{(A) } (1, 2) \quad \text{(D) } (-1, 1) \)
\( \text{(B) } (1, 4) \quad \text{(E) } (-1, 4) \)
\( \text{(C) } (2, 3) \)

319. Which statement best describes \( f \) at \( x = 0 ? \)
\( \text{(A) } f \text{ is a minimum} \)
\( \text{(B) } f \text{ is a maximum} \)
\( \text{(C) } f \text{ has a root} \)
\( \text{(D) } f \text{ has a point of inflection} \)
\( \text{(E) } \text{None of the above} \)

318. Which of the following is true based upon the graph above?
\( \text{(A) } f \text{ has a local maximum at } x = 0 \)
\( \text{(B) } f \text{ has a local maximum at } x = -1 \)
\( \text{(C) } f \text{ is a constant for } 1 < x < 4 \)
\( \text{(D) } f \text{ is decreasing for } -1 < x < 1 \)
\( \text{(E) } f \text{ is discontinuous at } x = 1 \)

151. The number of inflection points of \( f(x) = 4x^4 - 4x^2 \) is
\( \text{(A) } 0 \quad \text{(D) } 3 \)
\( \text{(B) } 1 \quad \text{(E) } 4 \)
\( \text{(C) } 2 \)
Given the function \( f(x) = e^{-x}(x^2 + 1) \)

1271. (a) For what values if \( f \) increasing?
(b) For what values is \( f \) concave up?
(c) Set up, but do not evaluate an integral (or set of integrals) which give the arc length of the segment of \( f \) from \( x = 0 \) to \( x = 1 \).

\[
\begin{align*}
(a) & \quad \{ \} \\
(b) & \quad x < 1 \text{ or } x > 3 \\
(c) & \quad \int_0^1 e^{-2x} (x-1)^4 + 1 \, dx
\end{align*}
\]

1268.

The figure above shows the graph of \( f' \), the derivative of a function \( f \). The domain of \( f \) is the set of all real numbers \( x \) such that \(-6 \leq x \leq 6 \).

(a) For what values of \( x \) does the graph of \( f \) have a horizontal tangent?
(b) For what values of \( x \) in the interval \(-6 < x < 6 \) does \( f \) have a relative minimum? Justify your answer.
(c) For what values of \( x \) is the graph of \( f \) concave upward?

\[
\begin{align*}
(a) & \quad x = \frac{11}{6}, -2, 0, 2, \frac{9}{2} \\
(b) & \quad x = \frac{11}{6}, 0, \frac{9}{2} \\
(c) & \quad x < \frac{7}{2} \text{ or } -\frac{1}{2} \leq x < 1 \text{ or } x > 3
\end{align*}
\]

Let \( f \) be a differentiable function, defined for all real \( x \), with the following properties:
1) \( f''(1) = f'(1) = f(1) \)
2) \( f \) is a polynomial of degree at most 2
3) \( f \) has only one fixed point (that is, there is exactly one \( p \) such that \( f(p) = p \)).
Find \( f(x) \).

\( f(x) = 0 \)

A particle moves along the \( x \)-axis in such a way that its acceleration at time \( t \) for \( t \geq 0 \) is given by \( a(t) = 6\sin(3t) \). At time \( t = 0 \), the velocity of the particle is \( v(0) = \sqrt{2} \) and its position is \( x(0) = 0 \).

(a) Write an equation for the velocity \( v(t) \) of the particle.
(b) Write an equation for the position \( x(t) \) of the particle.

\[
\begin{align*}
(a) & \quad v(t) = -2\cos(3t) + \sqrt{2} + 2 \\
(b) & \quad x(t) = -\frac{2}{3}\sin(3t) + t\sqrt{2} + 2t
\end{align*}
\]

A particle moves along the \( x \)-axis so that its velocity at any time \( t \geq 0 \) is given by \( v(t) = 1 - 2\cos \pi t \).

(a) Find the acceleration \( a(t) \) of the particle at any time \( t \).
(b) Find all values of \( t \), \( 0 \leq t \leq 4 \), for which the particle is at rest.
(c) Find the position \( x(t) \) of the particle at any time \( t \) if \( x(0) = 0 \).

\[
\begin{align*}
(a) & \quad a(t) = 2\pi \sin(\pi t) \\
(b) & \quad t = \frac{1}{3}, \frac{5}{3}, \frac{7}{3}, \frac{11}{3} \\
(c) & \quad x(t) = t - \frac{2\sin(\pi t)}{\pi}
\end{align*}
\]

Let \( f(x) = 8 - 2x^2 \) for \( x \geq 0 \) and \( f(x) \geq 0 \).

1264.

(a) The line tangent to the graph of \( f \) at the point \((x, y)\) intercepts the \( x \)-axis at \( x = 3 \). What are the coordinates of the point \((x, y)\)?
(b) An isosceles triangle whose base is the interval from \((0, 0)\) to \((k, 0)\) has its vertex on the graph of \( f \). For what value of \( k \) does the triangle have maximum area? Justify your answer.

\[
\begin{align*}
(a) & \quad \left(\sqrt{5} + 3, -12\sqrt{5} - 20\right) \\
(b) & \quad k = \sqrt{\frac{20}{3}}
\end{align*}
\]
III. INTEGRALS

2. Definite Integrals

1. Riemann Sums

729. Using a right Riemann sum, what is the area under the curve \( y = x^2 + x \) from \( x = 0 \) to \( x = 3 \) when \( n = 6 \)?

(A) 10.625  (D) 13.625
(B) 13.438  (E) 16.625
(C) 13.500

723. Using the Trapezoidal Rule, what is the area under the curve \( y = x^2 + x \) from \( x = 0 \) to \( x = 3 \) when \( n = 6 \)?

(A) 10.625  (D) 13.625
(B) 13.438  (E) 16.625
(C) 13.500

703. A Riemann sum to calculate the area under \( f(x) \) on the interval \( a \leq x \leq b \) with an infinite number of subintervals will yield the value

\[
\int_{a}^{b} f(x) \, dx
\]

(A) \( \int_{a}^{b} f(x) \, dx \)  (D) \( \frac{df(x)}{dx} \bigg|_{(b,a)} \)
(B) \( f(b) - f(a) \)  (E) \( \frac{d}{dx} \int_{a}^{b} f(x) \, dx \)
(C) \( \frac{df(x)}{dx} \bigg|_{(a,b)} \)

658. What is an approximation for the area under the curve \( y = 4x - x^2 \) on the interval \( [0, 4] \) using the midpoint formula with 20 subintervals?

(A) 10.589  (D) 10.681
(B) 10.624  (E) 10.703
(C) 10.667

657. What is an approximation for the area under the curve \( y = \frac{1}{x} \) on the interval \( [2, 5] \) using the trapezoidal rule with 9 subintervals?

(A) 0.868  (D) 0.918
(B) 0.915  (E) 0.968
(C) 0.916

654. What is an approximation for the area under the curve \( y = \frac{3}{1 + x^2} \) on the interval \( [0, 3] \) using the trapezoidal rule with 5 subintervals?

(A) 2.932  (D) 3.750
(B) 3.742  (E) 4.552
(C) 3.747

651. What is an approximation for the area under the curve \( y = \sqrt{5 + x^2} \) on the interval \( [0, 3] \) using the midpoint formula with 4 subintervals?

(A) 11.992  (D) 17.059
(B) 16.155  (E) 22.126
(C) 16.456

647. Using a left Riemann sum, what is the area under the curve \( y = \sqrt{x} \) from \( x = 1 \) to \( x = 3 \) when \( n = 4 \)?

(A) 2.976  (D) 2.793
(B) 2.800  (E) 2.610
(C) 2.797

460. Using a left Riemann sum, what is the area under the curve \( y = 2x - x^3 \) from \( x = 1 \) to \( x = 2 \) when \( n = 4 \)?

(A) 0.53125  (D) 0.67187
(B) 0.65625  (E) 0.78125
(C) 0.66667
III. INTEGRALS

4. Applications

1847. The area of the region bounded by the graph of $y = 2xe^x$ and the $x$-axis from $x = 0$ to $x = 1$ is
(A) $e^2 - 1$  (D) $e$
(B) $e - 1$  (E) $e^3(1 - e)$
(C) $e^2 - e$

Let $f(x) = x\sin(x - \pi)$, $0 < x < 2\pi$. The total area bounded by $f(x)$ and the $x$-axis on the interval $0 < x < 2\pi$ is
(A) $\pi$  (D) $4\pi$
(B) $2\pi$  (E) $5\pi$
(C) $3\pi$

The area of the region enclosed by $y = 2x^2 - 4$ and $y = \sqrt{9 - x^2}$ is
(A) $8.368$  (D) $15.538$
(B) $9.082$  (E) $16.736$
(C) $12.874$

847. What is the average area of all squares with sides between 3 and 12?
(A) 63  (D) 69
(B) 65  (E) 71
(C) 67

848. What is the ratio of the area of the rectangle to the shaded part of it above $y = x^2$?
(A) $2 : 1$  (D) $4 : 3$
(B) $3 : 1$  (E) $5 : 4$
(C) $3 : 2$

A. Area

1. Area

1674. If \( \int_0^1 (2x^2 - x + 3)\,dx \) is approximated by three inscribed rectangles of equal width on the $x$-axis, the approximation is
(A) $16$  (D) $23.5$
(B) $22$  (E) $31$
(C) $22.5$

Find the area in the first quadrant bounded by the graphs of $y = \cot x$, $y = 2\sin x$, and the $x$-axis.
(A) $0.438$  (D) $0.909$
(B) $0.470$  (E) $1.571$
(C) $0.746$

The total area enclosed between the graphs of $y = 2\cos x$ and $y = \frac{x}{2}$ is
(A) $3.862$  (D) $4.812$
(B) $3.985$  (E) $5.914$
(C) $4.547$

1772. If \( \int_a^b f(x)\,dx = -2 \) and \( \int_a^b g(x)\,dx = -5 \), which of the following must be true?
I. $f(x) > g(x)$ for $a \leq x \leq b$
II. $\int_a^b [f(x) - g(x)]\,dx = 3$
III. $\int_a^b [f(x)g(x)]\,dx = 10$
(A) I only  (D) II and III
(B) II only  (E) I, II and III
(C) I and II

The area in the first quadrant bounded by the graphs of $y = x^2 + 3$ and $y = 12$ is
(A) $9$  (D) $36$
(B) $18$  (E) $30$
(C) $24$

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The base of a solid is the region bounded by the line \(-y + 6 = 2x\) and the coordinate axes. What is the volume of the solid generated if every cross section perpendicular to the \(x\)-axis is a semicircle?

(A) \(\frac{7}{2} \pi\)  
(B) \(\frac{9}{2} \pi\)  
(C) \(9\pi\)

The region bounded by the graphs of \(y = 3 - e^{-x}\) and \(y = x^2\) is revolved around the \(x\)-axis. The volume of the resulting solid is

(A) 9.805  
(B) 11.473  
(C) 19.975  
(D) 27.362  
(E) 30.805

What is the volume of the solid when the region bounded by \(xy = 6\) and \(x + y = 5\) is revolved about the \(x\)-axis?

(A) \(\pi\)  
(B) \(\frac{2}{3} \pi\)  
(C) \(\frac{1}{3} \pi\)

What is the volume of the solid when the region bounded by \(y = \frac{x}{3}, y = x, y = 1\) and \(y = 2\) is revolved about the \(x\)-axis?

(A) \(8\pi\)  
(B) \(\frac{28}{3} \pi\)  
(C) \(\frac{32}{3} \pi\)

What is the volume of the solid when the region bounded by \(y = \cos x, x = \frac{\pi}{2}\) and \(y = 0\) is revolved about the \(y\)-axis?

(A) \(1\)  
(B) \(\pi\)  
(C) \(2\pi\)

What is the volume of the solid when the region bounded by \(x = \sqrt{3} + y, x = 0\) and \(y = 6\) is revolved about the \(y\)-axis?

(A) \(\frac{23}{2} \pi\)  
(B) \(54\pi - \pi \sqrt{3}\)  
(C) \(4\pi \sqrt{6} - 2\pi \sqrt{3}\)

966. What is the volume of the solid when the region bounded by \(y = x^{3/2}, x = 0, x = 4\) and \(y = 0\) is revolved about the \(x\)-axis?

(A) \(1028\pi\)  
(B) \(64\pi\)  
(C) \(3\pi\)

941. What is the volume of the solid that has a circular base of radius \(r\) and every plane section perpendicular to a diameter is semi-circle?

(A) \(\frac{1}{2} \pi r^3\)  
(B) \(\frac{1}{2} \pi r^3\)  
(C) \(\frac{1}{2} \pi r^3\)

940. What is the volume of the solid that has a circular base of radius \(r\) and every plane section perpendicular to a diameter is an isosceles triangle?

(A) \(\frac{2}{3} \pi r^3\)  
(B) \(\frac{2}{3} \pi r^3\)  
(C) \(\pi r^3\)

939. What is the volume of the solid that has a circular base of radius \(r\) and every plane section perpendicular to a diameter is an equilateral triangle?

(A) \(r^3 \sqrt{3}\)  
(B) \(2r^3 \sqrt{3}\)  
(C) \(4r^3 \sqrt{3}\)

922. What is the volume of the solid when the region bounded by \(y = \sqrt{x}, y = x\) is revolved about the \(x\)-axis?

(A) \(\pi\)  
(B) \(2\pi\)  
(C) \(\frac{1}{6} \pi\)

917. What is the volume of the solid that has a circular base of radius \(r\) and every plane section perpendicular to a diameter is a square?

(A) \(2\pi r^3\)  
(B) \(4\pi r^3\)  
(C) \(\frac{8}{3} r^3\)
III. INTEGRALS
C. Differential Equations

4. Applications
1. Differential Equations

1046. If the population of a city increases continuously at a rate proportional to the population at that time and the population doubles in 20 years, then what is the ratio of the population after 60 years to the initial population?

(A) 3:1  (D) 9:2
(B) 3:2  (E) 9:4
(C) 9:1

Which of the following curves passes through the point (1,1) and whose slope at any point is equal to $y^2x$?

(A) $y = \frac{2}{3 - x^2}$  (D) $y = \frac{3}{2 - x^2}$
(B) $y = \frac{3}{x^2 - 2}$  (E) $y = \frac{2}{x^2 - 3}$
(C) $y = -x^2$

The general solution to the differential equation $dy/dx = 2$ is a family of

(A) parabolas  (D) ellipses
(B) straight lines  (E) circles
(C) hyperbolas

The general solution to the differential equation $dy/dx = x$ is a family of

(A) parabolas  (D) ellipses
(B) straight lines  (E) circles
(C) hyperbolas

1027. If $\frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}}$ and $y = 2$ when $x = \sqrt{5}$, what is the solution to the differential equation?

(A) $y = \sqrt{1+x^2} + 1$  (D) $y = \sqrt{1+x^2}$
(B) $y = \frac{1}{2} \sqrt{1+x^2} + 1$  (E) $y = 2\sqrt{1+x^2}$
(C) $y = \frac{1}{2} \sqrt{1+x^2}$

1023. If $\frac{dy}{dx} = \frac{1}{2} e^y$ and $y = 2$ when $x = 2$, what is the solution to the differential equation?

(A) $y = \ln|\frac{1}{2}| + 2$  (D) $y = \frac{1}{2} \ln|x| + 2$
(B) $y = \ln|\frac{3}{2}| + 2$  (E) $y = \ln|x| + 2$
(C) $y = 2\ln|x| + 2$

1022. If $\frac{dy}{dx} = \frac{y}{\sqrt{x}}$ and $y = 1$ when $x = 1$, what is the solution to the differential equation?

(A) $y = e^{\sqrt{x} - 2}$  (D) $y = e^{\sqrt{x} - 1}$
(B) $y = e^{\sqrt{x}}$  (E) $y = e\sqrt{x} - 1$
(C) $y = \sqrt{x}e^{\sqrt{x}}$

What is a solution to the differential equation $ydy = 2xdx$?

(A) $x^2 + y^2 = 4$  (D) $x^2 = 2 - y^2$
(B) $y^2 = x^2$  (E) $x^2 - y^2 = 2$
(C) $y^2 - 2x^2 = 0$

965. A growth rate of 5% per year is equal to a continuous growth rate of

(A) $\ln(0.95)$  (D) $0.95\ln(1.05)$
(B) $\ln(1.05)$  (E) $\ln(1.05) - \ln(0.95)$
(C) $1.05\ln(0.05)$
A particle moves along a curve so that at any time \( t \geq 0 \) its velocity is given by \( v(t) = \ln (t + 1) - t^2 + 1 \). The total distance traveled by the particle from \( t = 1 \) to \( t = 3 \) is

(A) 3.986  
(B) 4.289  
(C) 4.508

The acceleration of a car traveling on a straight track along the \( x \)-axis is given by the equation \( a(t) = 2t + 1 \), where \( a \) is in meters per second squared and \( t \) is in seconds. If at \( x(0) = 0 \) and \( v(0) = 0 \), what is its displacement at \( t = 3 \)?

(A) 1 m  
(B) 7 m  
(C) 9 m  
(D) 12 m  
(E) 13.5 m

The acceleration of a car traveling on a straight track along the \( y \)-axis is given by the equation \( a = 5 \), where \( a \) is in meters per second squared and \( t \) is in seconds. If at \( t = 0 \) the car’s velocity is 3 m/s, what is its velocity at \( t = 2 \)?

(A) 5 m/s  
(B) 3 m/s  
(C) 10 m/s  
(D) 13 m/s  
(E) 15 m/s

The velocity of a particle is given by the equation \( v(t) = 3t^2 - 4 \). If \( s(0) = 2 \), then what is the position function of the particle?

(A) \( s(t) = 3t^2 + 2 \)  
(B) \( s(t) = \frac{3}{2}t^2 - 4t + 2 \)  
(C) \( s(t) = t^2 - 4t + 2 \)  
(D) \( s(t) = 3t^2 + 2 \)  
(E) \( s(t) = t^2 - 4t + 2 \)

The equation \( v(t) = 3t^2 - 4t + 2 \), where \( v \) is in meters per second and \( t \) is in seconds gives the velocity of a vehicle moving along a straight track. The vehicle’s initial position is \( y = -1 \) m. At what time does the car pass the origin?

(A) 0 s  
(B) 1 s  
(C) 2 s  
(D) 3 s  
(E) 4 s

A particle moves along a path so that at any time \( t \) its acceleration is given by \( a(t) = 2t + 1 \). At time \( t = 0 \), its velocity is \( v(0) = -6 \). For what value(s) of \( t \) is the particle at rest?

(A) 0 only  
(B) 2 only  
(C) –3 only  
(D) 2 and –3  
(E) No values

A particle moves along a path so that’s its velocity is given by \( v(t) = t^2 - 4 \). How far does the particle travel from \( t = 0 \) to \( t = 4 \)?

(A) \( \frac{16}{3} \)  
(B) 8  
(C) 16  
(D) 16.819  
(E) 20

The equation \( v(t) = 3t^2 - 4t + 2 \), where \( v \) is in meters per second and \( t \) is in seconds gives the velocity of a vehicle moving along a straight track. The vehicle’s initial position is 3 meters. What distance has the vehicle traveled after 4 seconds?

(A) 30 m  
(B) 31 m  
(C) 32 m  
(D) 33 m  
(E) 34 m

A particle travels in a straight line with a constant acceleration of 2 meters per second per second (m/s²). If the velocity of the particle is 5 meters per second at the time \( t = 1 \) second, how far does the particle travel from \( t = 1 \) to \( t = 3 \)?

(A) 7 m  
(B) 8 m  
(C) 10 m  
(D) 11 m  
(E) 14 m

A particle travels along the \( x \)-axis with velocity at time \( t \), \( v(t) = \cos(t^2) \). If at time \( t = 0 \) the particle is at \( x(0) = 2 \), where is the particle at \( t = 2 \)?

(A) 0.492  
(B) 1.529  
(C) 2.982  
(D) 2.461  
(E) It cannot be determined from the information given.
For time \( t \), \( 0 \leq t \leq 2\pi \), the position of a particle, is given by \( x = \sin^2 t \) and \( y = e^t \cos t \).

(a) Find the formula for the slope of the path of the particle as a function of time.
(b) For what \( t \) is the line tangent to the curve vertical.
(c) Set up an integral for the distance traveled by the particle from \( t = 0 \) to \( t = 1 \).

\[
\begin{align*}
(a) \quad & s(t) = \frac{e^t (\cos t - \sin t)}{2 \sin t \cos t} \\
(b) \quad & 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi \\
(c) \quad & \int_0^1 \sqrt{\sin^2 2t + e^{2t} (\cos t - \sin t)^2} \, dt
\end{align*}
\]

Let \( f \) be function given by \( f(x) = x^2 \) and let \( g \) be the function given by \( g(x) = kx - 4 \), where \( k \) is a positive constant such that \( g \) is tangent to the graph of \( f \).

(a) Find the value of \( k \).
(b) Find the area bounded on top by the line perpendicular to \( g \) and on the bottom by \( f(x) \).
(c) Find the volume of the solid generated by revolving the region from part (b) about the line \( y = 0 \).

\[
\begin{align*}
(a) \quad & k = 4 \\
(b) \quad & 44.667 \\
(c) \quad & 385.36
\end{align*}
\]

Consider \( \int_0^5 \frac{1}{1 + x^4} \, dx \).

(a) Require the denominator as \( (x^2 + 1)^2 - 2x^2 \).
(b) Split up the denominator into a product of linear factors.
(c) Integrate by partial fractions.

Let \( R \) be enclosed by the graph of \( y = x \ln x \), the line \( x = 2 \), and the \( x \)-axis.

(a) Find the net area of region \( R \).
(b) Find the volume of the solid generated by revolving region \( R \) about the \( x \)-axis.
(c) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated by revolving region \( R \) about the line \( x = 2 \).

\[
\begin{align*}
(a) \quad & A = 2 \ln 2 - 1 \\
(b) \quad & V = \pi \left[ \frac{8}{3} \left( \ln 2 \right)^2 - \frac{16}{9} \ln 2 + \frac{14}{27} \right] \\
(c) \quad & V = 2\pi \int_1^2 (2-x) x \ln x \, dx
\end{align*}
\]

Let \( f \) be a function that is defined for all real numbers \( x \) and that satisfies the following properties.

\[
\begin{align*}
(\text{i}) \quad & f''(x) = 10x - 12 \\
(\text{ii}) \quad & f'(1) = -16 \\
(\text{iii}) \quad & f(0) = 8
\end{align*}
\]

(a) Find all values of \( x \) such that the line tangent to the graph at \((x, f(x))\) is horizontal.
(b) Find \( f(x) \).
(c) Find the average value of \( f' \) on the interval \( 2 \leq x \leq 5 \).

\[
\begin{align*}
(a) \quad & -6.3 \\
(b) \quad & f(x) = \frac{5x^3}{3} - 6x^2 - 9x + 8 \\
(c) \quad & \frac{232}{3}
\end{align*}
\]

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IV. POLYNOMIAL APPROXIMATIONS AND SERIES

A. Convergence and Divergence

1. Series of Constants

1177. The series \( \sum_{n=1}^{\infty} \left( \frac{1}{5^n} \right) \) is

(A) convergent and decreasing
(B) convergent and increasing
(C) divergent and decreasing
(D) divergent and increasing
(E) divergent and remain the same

1174. The series \( \sum_{n=1}^{\infty} \left( \frac{2^n}{1 + 2^n} \right) \) is

(A) neither increasing nor decreasing
(B) decreasing and convergent
(C) decreasing and divergent
(D) increasing and convergent
(E) increasing and divergent

Which one of the following series is divergent?

1159. (A) \( \sum_{k=1}^{\infty} k^{-\frac{1}{2}} \)
(B) \( \sum_{k=1}^{\infty} \frac{1}{k^\pi} \)
(C) \( \sum_{k=1}^{\infty} \frac{1}{k} \)

All of the following are examples of a geometric series except

1183. For what values of \( n \) is the series \( \sum_{n=1}^{\infty} \left( \frac{8^n}{n!} \right) \) decreasing?

(A) \( n \geq 8 \)
(B) \( n \leq 8 \)
(C) \( n \geq 7 \)

Which of the following series is convergent?

1148. (A) \( \sum_{n=1}^{\infty} \frac{\ln k}{9k} \)
(B) \( \sum_{n=1}^{\infty} \frac{1}{k + 9} \)
(C) \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{k + 9}} \)

Which of the following infinite series has increasing terms?

1143. (A) \( \sum_{n=1}^{\infty} \left( n - 2^n \right) \)
(B) \( \sum_{n=1}^{\infty} \frac{n^n}{n!} \)
(C) \( \sum_{n=1}^{\infty} \left( \frac{n}{1 - 2n} \right) \)

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IV. POLYNOMIAL APPROXIMATIONS AND SERIES

A. Taylor Series

2. Taylor Series

1090. Which of the following generate Taylor series?

(A) \( f(x) = \frac{1}{1-x} \) about 0

(B) \( f(x) = \ln(x-1) \) about 1

(C) \( f(x) = \sqrt{x-2} \) about 2

(D) \( f(x) = \sqrt{1+x} \) about -1

(E) \( f(x) = \tan x \) about \( \frac{\pi}{2} \)

1088. For what values of \( x \) does the following power series converge?

\[ \sum_{n=1}^{\infty} \frac{x^n}{n} \]

(A) \(-1 < x < 1\)  (D) \(-1 < x \leq 1\)

(B) \(-1 \leq x \leq 1\)  (E) \(x > 1\) or \(x < 1\)

(C) \(-1 \leq x < 1\)

1044. For what values of \( x \) does the following power series diverge?

\[ \sum_{n=1}^{\infty} \frac{(x+3)^n}{n!} \]

(A) \(-3 \leq x < 3\)  (D) \(-3 < x < 3\)

(B) \(-3 \leq x \leq 3\)  (E) \{\}

(C) \(-3 < x \leq 3\)

1031. What is the 3rd order Taylor polynomial at \( x = 0 \) for \( f(x) = \sin x \)?

(A) \( x + \frac{x^3}{3!} \)  (D) \( x - \frac{x^3}{3!} \)

(B) \( x - \frac{x^2}{2!} + \frac{x^3}{3!} \)  (E) \( x + \frac{x^2}{2!} + \frac{x^3}{3!} \)

(C) \( x + \frac{x^2}{2!} - \frac{x^3}{3!} \)

1026. What is the 2nd order Taylor polynomial at \( x = \pi \) for \( f(x) = \cos x \)?

(A) \(-1 + (x-\pi) - \frac{(x-\pi)^2}{2!} \)

(B) \(1 - \frac{(x-\pi)^2}{2!} \)

(C) \(-1 + \frac{(x-\pi)^2}{2!} \)

(D) \(x-\pi + \frac{(x-\pi)^2}{2!} \)

(E) \(1 - (x-\pi) + \frac{(x-\pi)^2}{2!} \)

1021. What is an approximation for \( \ln(0.7) \) using the first three terms of the Taylor series \( f(x) = \ln(1+x) \) about \( x = 0? \)

(A) -0.340  (D) -0.355

(B) -0.349  (E) -0.357

(C) -0.354